PC3231 Electricity and Magnetism 2 (Semester 1: AY 2011-12)

I. Elecric field in matter

(i) Applying the Gauss's law in dielectic, $D = \sigma$ in each slab. Note that $D = 0$ inside the metal parallel plate.

(ii) Dielectric constant is another name for relative permittivity ϵ_r. Using $D = \epsilon E$ and $\epsilon = \epsilon_r \epsilon_0$, $E_1 = \frac{\sigma}{2\epsilon_0}$ for slab 1 and $E_2 = \frac{2\sigma}{3\epsilon_0}$ for slab 2.

(iii) Using $P = \epsilon_0 \chi_e E$ and $\chi_e = 1 + \epsilon_r$, $P_1 = \frac{\sigma}{2}$ for slab 1 and $P_2 = \frac{\sigma}{3}$ for slab 1.

(iv) Potential difference $V = E_1 a + E_2 a = \frac{7\sigma a}{6\epsilon_0}$.

(v) Volume bound charge density $\rho_b = 0$ as the polarisation is uniform. At the top of slab 1, $\sigma_b = -\frac{\sigma}{2}$; at the bottom of slab 1, $\sigma_b = \frac{\sigma}{2}$. At the top of slab 2, $\sigma_b = -\frac{\sigma}{3}$; at the bottom of slab 2, $\sigma_b = \frac{\sigma}{3}$.

II. Stress and momentum

(i) $E_x = 0$, $E_y = 0$ and $E_z = -\frac{\sigma}{\epsilon_0}$; $B_x = 0$, $B_y = 0$ and $B_z = 0$. Elements of the stess tensor: $T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij}B^2\right)$, where i and j can be either x, y or z. Hence,

$$T = \frac{\sigma^2}{2\epsilon_0}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(ii) As $B = 0$,$S = 0$. Thus, integrating over the xy plane: $d\mathbf{a} = -dxdy\hat{\mathbf{z}}$ (negative because outward with respect to a surface enclosing the top plate). Thus, $F_z = \int T_{zz} da_z = -\frac{\sigma^2}{2\epsilon_0}A$. The force per unit area is $\mathbf{f} = \frac{\mathbf{F}}{A} = -\frac{\sigma^2}{2\epsilon_0}\hat{\mathbf{z}}$.

(iii) $\frac{\sigma^2}{2\epsilon_0}$is the momentum per unit area, per unit time, crossing the xy plane.

III. Rectangular waveguide

(i) Let a as the length of the longer side of the rectangular waveguide. For TE_{10}, $E_z = 0$, $B_z = B_0 \cos\left(\frac{\pi x}{a}\right)$, $E_x = 0$, $B_x = \frac{-ik}{\left(\frac{\omega}{c}\right)^2 - k^2}\frac{\pi}{a}B_0 \sin\left(\frac{\pi x}{a}\right)$, $E_y = \frac{i\omega}{\left(\frac{\omega}{c}\right)^2 - k^2}\frac{\pi}{a}B_0 \sin\left(\frac{\pi x}{a}\right)$ and $B_y = 0$.

(ii) From part (i), $\mathbf{E} = \{0\,,\, E_y \exp\left[i\left(kz - \omega t\right)\right]\,,\, 0\}$and. Also, we have found that$\mathbf{B} = \{B_x \exp\left[i\left(kz - \omega t\right)\right]\,, 0,\, B_z \exp\left[i\left(kz - \omega t\right)\right]\}$. Thus, $\langle \mathbf{S} \rangle = \frac{1}{2\mu_0}\left[\mathbf{E} \times \mathbf{B}^\star\right] = \hat{\mathbf{k}} \frac{\omega k \pi^2 B_0^2}{2\mu_0\left[\left(\frac{\omega}{c}\right)^2 - k^2\right]^2} \left[\left(\frac{1}{a}\right)^2 \sin^2\left(\frac{\pi x}{a}\right)\right]$.

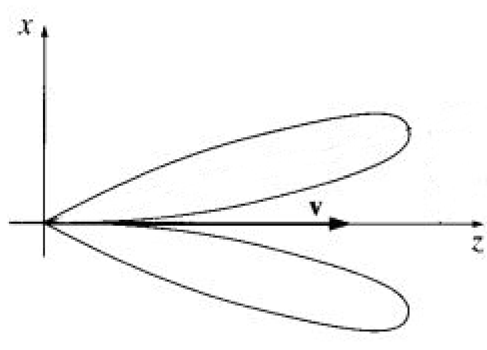
(iii) For certainTE$_{mn}$ mode, the group velocity is given by $v_g = c\sqrt{1 - \left(\frac{\omega_{mn}}{\omega}\right)^2}$ and the cutoff frequency is found using$\omega_{mn} = c\pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$. It is given that the length of the longer side $a = 2.28$cm, length of the shorter side is $b = 1.01$cm and the driving frequency is $\omega = 2 \times 10^{10}$Hz. The first three TE modes are TE$_{10}$, TE$_{01}$ and TE$_{11}$. The corresponding group velocities are 2.83×10^8m/s, 2.00×10^8m/s and 1.75×10^8m/s.

IV. Bremsstrahlung Radiation

(i) The angular distribution of the power radiated by a point charge is given by $\frac{dp}{d\Omega} = \frac{q^2}{16\pi^2\epsilon_0} \frac{|\hat{\mathbf{r}} \times (\mathbf{u} \times \mathbf{a})|^2}{(\hat{\mathbf{r}} \cdot \mathbf{u})^5}$ where $u = \hat{\mathbf{r}}c - \mathbf{v}$ and $\hat{\mathbf{r}}$ is the vector from the point charge to the observer. If v and a is instantaneously collinear along the z direction, $\mathbf{u} \times \mathbf{a} = c\,(\hat{\mathbf{r}} \times \mathbf{a})$. Now, $|\hat{\mathbf{r}} \times (\mathbf{u} \times \mathbf{a})\,|^2 = c\hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \mathbf{a}) = (\hat{\mathbf{r}} \cdot \mathbf{a})\,\hat{\mathbf{r}} - \mathbf{a}$. Thus, $|\hat{\mathbf{r}} \times (\mathbf{u} \times \mathbf{a})\,|^2 = a^2 - (\hat{\mathbf{r}} \cdot \mathbf{a})^2 = a^2\left(1 - \sin^2\theta\right) = a^2\cos^2\theta$, where θ is the angle between the $\hat{\mathbf{r}}$ and $\mathbf{a}$. If we let $\beta = \frac{v}{c}$, we will find that $\frac{dp}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\sin^2\theta}{(1 - \beta\cos\theta)^5}$.

(ii) The total power $P = \int \frac{dp}{d\Omega} d\Omega = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \frac{\sin^2\theta}{(1 - \beta\cos\theta)^5}$. If we let $x = \cos\theta$, $P = \frac{\mu_0 q^2 a^2}{8\pi c} \int_{-1}^{1} dx \frac{\left(1 - x^2\right)}{(1 - \beta x)^5}$. Using integration by parts, we will get $P = \frac{\mu_0 q^2 a^2}{6\pi c}\gamma^6$, where $\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$.

(iii)



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