NATIONAL UNIVERSITY OF SINGAPORE

PC3231 Electricity and Magnetism 2

(Semester I: AY 2011-12)

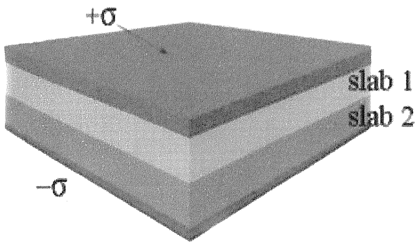
Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **4** questions and comprises **4** printed pages.
2. Answer any **3** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. One Help Sheet (A4 size, both sides) is allowed for this examination.

1. <u>Electric field in matter</u>

The space between the plates of a parallel plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is 2a.



Slab 1 has a dielectric constant of 2. Slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is +σ, and on the bottom plate is −σ.

Determine

(i) the electric displacement **D** in each slab.
(ii) the electric field **E** in each slab.
(iii) the polarization **P** in each slab.
(iv) the potential difference between the plates.
(v) the location and amount of all bound charges.

2. <u>Stress and momentum</u>

Consider an infinite parallel plate capacitor, with lower plate (at $z= -d/2$) carrying the charge density −σ, and the upper plate (at $z=+d/2$) carrying the charge density +σ.

(i) Determine all nine elements of the stress tensor in the region between the plates. Display your answer as a 3x3 matrix:

$$\begin{vmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{vmatrix}$$

(ii) Using the equation

$$\mathbf{F} = \oint_S \overleftrightarrow{\mathbf{T}} \cdot d\mathbf{a} - \varepsilon_0 \mu_0 \frac{d}{dt} \oint_v \mathbf{S}\, d\tau,$$

show that the force per unit area **f** on the top plate is

$$\mathbf{f} = \frac{\mathbf{F}}{A} = -\frac{\sigma^2}{2\varepsilon_0}\hat{\mathbf{z}}.$$

(iii) What is the momentum per unit area, per unit time, crossing the xy plane?

3. <u>Rectangular waveguide</u>

Consider the TE_{10} mode of a rectangular waveguide propagating in the z direction.

(i) What are the components (E_x, E_y) and (B_x, B_y, B_z) of the electric and magnetic fields for the TE_{10} mode? You are given that

$$E_x = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)$$

$$E_y = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right)$$

$$E_z = 0$$

and

$$B_x = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

$$B_y = \frac{i}{\left(\omega/c\right)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

$$B_z = B_0 \cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

(ii) Find the time averaged Poynting vector $\langle \mathbf{S} \rangle$ of the TE_{10} mode in the waveguide.

(iii) Consider the X-band rectangular waveguide of cross-sectional dimensions 2.28 cm X 1.01 cm. Determine the group velocities of propagation for the first three TE modes in this waveguide when the driving frequency is 2 X 10^{10} Hz.

4. Bremsstrahlung radiation

The power P radiated by a point charge q in arbitrary motion with velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ into a patch of area $r^2 d\Omega$ is given by the general expression

$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2 \varepsilon_0} \frac{1}{(\hat{\mathbf{r}} \cdot \mathbf{u})^5} \left[\, | \, \hat{\mathbf{r}} \times (\mathbf{u} \times \mathbf{a}) \, |^2 \right]$$

where $\mathbf{u} = \hat{\mathbf{r}}c - \mathbf{v}$ and $\mathbf{r}$ is the vector from the point charge to the observer.

(i) Suppose $\mathbf{v}$ and $\mathbf{a}$ are instantaneously collinear at retarded time t_r, and take the z axis to point along $\mathbf{v}$, show that the angular distribution of the power radiated is given by

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\sin^2\theta}{(1 - \beta\cos\theta)^5}$$

(ii) Show that the total power radiated P is given by

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \gamma^6$$

where $\gamma = 1/\sqrt{(1 - v^2/c^2)}$

(iii) Sketch the angular dependence of radiated power for $v \ll c$ and for very large v.

~ End of Paper

TSH