NATIONAL UNIVERSITY OF SINGAPORE

PC3232 Nuclear and Particle Physics

(Semester I: AY 2012-13)

Time Allowed: 2 Hours

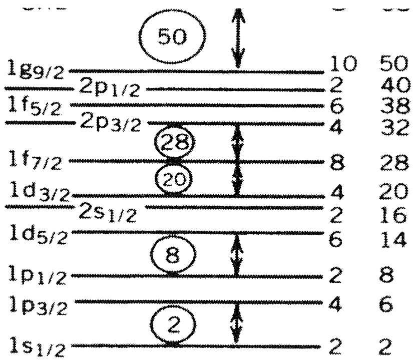
INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **5** questions and comprises **12** printed pages.
2. Answer **ALL** questions.
3. Answers to the questions are to be written on this examination booklet.
4. This is a CLOSED BOOK examination.
5. A help sheet (A4-size, two-sided, hand-written) is allowed for the examination.

Matriculation Number: ____________________________

Question	Marks
1	/25
2	/15
3	/15
4	/30
5	/15
Total	/100

1. The shell model predicts the following energy level distribution.



(a) Explain in reasonable detail the principle underlying the shell model of the nucleus and how this model is able to predict the magic numbers 2, 8, 20, 28, 50, 82, 126, 184. [10 marks]

(b) The ground state of a nucleus with odd proton and odd neutron is determined by the coupling of proton and neutron shell-model state: $\vec{I} = \vec{j}_p + \vec{j}_n$. Consider the following nuclei with their spin parity assignment: ${}^{16}_{7}N_9 \rightarrow 2^-$; ${}^{12}_{5}B_7 \rightarrow 1^+$; ${}^{34}_{15}P_{19} \rightarrow 1^+$; ${}^{28}_{13}Al_{15} \rightarrow 3^+$.

(i) Compute the odd proton and odd neutron shell model states, and draw simple vector diagrams (for $\vec{j}_p$ and $\vec{j}_n$) to illustrate these couplings. [4 marks]

(ii) Replace $\vec{j}_p$ and $\vec{j}_n$ by $\vec{\ell}_p + \vec{s}_p$ and $\vec{\ell}_n + \vec{s}_n$ respectively. [4 marks]

(iii)Examine your four diagrams and deduce an empirical rule for the relative orientation of $\vec{s}_p$ and $\vec{s}_n$ in the ground state. [3 marks]

(iv)Use your empirical rule to predict the spin-parity assignments for ${}^{26}_{11}Na_{15}$ and ${}^{28}_{11}Na_{17}$. [4 marks]

${}^{26}_{11}Na_{15} : I =$ _______ ; ${}^{28}_{11}Na_{17} : I =$ _______

2. A beam of negative pions π^- impinges on a stationary proton target. K^- is produced in this strong process. This reaction is given by $\pi^- + p \rightarrow K^- + X + Y$.

   (a) Deduce the **two** possible identities for unknown X and Y, and indicate the conservation laws used. [5 marks]

   Case 1: $X =$ _______ ; $Y =$ _______

   Case 2: $X =$ _______ ; $Y =$ _______

   (b) From first principles, derive an expression for the threshold laboratory kinetic energy of π^- for the K^- production. (From the above two cases, you only need to consider the case which gives the lower threshold energy. Mass of the various particles can be found in the last page.) [10 marks]

3. Consider the following fusion processes.

Reaction 1: ${}^{2}H+{}^{2}H\rightarrow{}^{4}He+\gamma$

Reaction 2: ${}^{2}H+{}^{2}H\rightarrow{}^{3}He+n$

Reaction 3: ${}^{2}H+{}^{2}H\rightarrow{}^{3}H+p$

Among the above nuclei, ${}^{4}He$ is the most stable. However, it is observed that reaction 2 and 3 are more likely to occur than reaction 1. Why? Discuss. [15 marks]

You may use the following binding energies.

Nucleus	${}^{4}He$	${}^{3}He$	${}^{3}H$	${}^{2}H$
Binding energy(MeV)	28.2957	7.7180	8.4818	2.2246

4. (a) Describe in reasonable detail the physical principle underlying the detection of a gamma ray using a scintillation detector and a photo-multiplier tube, including the various interactions between the gamma ray and the scintillator. [15 marks]

Question 4 (a) answer continues …

(b) A nucleus has the following sequence of states beginning with the ground state $\frac{3}{2}^{+}$, $\frac{5}{2}^{+}$, $\frac{1}{2}^{-}$ and $\frac{3}{2}^{-}$.

(i) Draw an energy level scheme showing all possible γ transitions and their multipole assignments.

(ii) Indicate the dominant transitions for every allowed transition. [15 marks]

5. Consider the following non-leptonic (hadronic) weak decays.

$K^0 \to \pi^+ + \pi^-$; $\Xi^- \to \Lambda^0 + \pi^-$; $\Lambda^0 \to n + \pi^0$; $\Sigma^+ \to p + \pi^0$

(a) Deduce the possible selection rules for strangeness number ΔS and 3rd component of isospin ΔI_3 for the above non-leptonic decays. [4 marks]

$\Delta S =$ _____ ; $\Delta I_3 =$ _____

(b) Similarly, deduce the possible selection rules for strangeness number ΔS and 3rd component of isospin ΔI_3 for a particular class of semi-leptonic decays below. [4 marks]

$\pi^+ \to \pi^0 + e^+ + \nu_e$; $n \to p + e^- + \overline{\nu}_e$; $\pi^- \to \mu^- + \overline{\nu}_\mu$; $\Sigma^- \to \Lambda^0 + e^- + \overline{\nu}_e$

$\Delta S =$ _____ ; $\Delta I_3 =$ _____

(c) Hence, or otherwise discuss whether the decay $\Xi^- \to n + \pi^-$ is allowed or forbidden. [3 marks]

(d) Draw a possible Feynman diagram at quark level for the decay $\Xi^- \rightarrow \Lambda^0 + \pi^-$. [4 marks]

Quark content and masses (MeV/c^2)

Mesons

$\pi^- = d\bar{u}$ (139.570) ; $\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$ (134.977) ; $\pi^+ = u\bar{d}$ (139.570)

$K^- = s\bar{u}$ (493.667) ; $\bar{K}^0 = s\bar{d}$ (497.614) ; $K^0 = d\bar{s}$ (497.614) ; $K^+ = u\bar{s}$ (493.667)

Baryons

$n = udd$ (939.565) ; $p = uud$ (938.272) ; $\Delta^- = ddd$; $\Delta^0 = udd$; $\Delta^+ = uud$; $\Delta^{++} = uuu$

(1232 for all Δ)

$\Lambda^0 = uds$ (1115.683) ; $\Sigma^- = dds$ (1197.449) ; $\Sigma^0 = uds$ (1192.642) ; $\Sigma^+ = uus$ (1189.37)

$\Xi^- = dss$ (1321.71) ; $\Xi^0 = uss$ (1314.86)

$\Omega^- = sss$ (1672.45)

Gell-Mann-Nishijima relation $Q = I_3 + \frac{1}{2}(A + S)$ where Q is the charge, I_3 is the 3rd component of isospin, A is the baryon number and S is the strangeness number.

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