NATIONAL UNIVERSITY OF SINGAPORE

PC3232 Nuclear and Particle Physics

(Semester I: AY 2014-15)

Time Allowed: 2 Hours

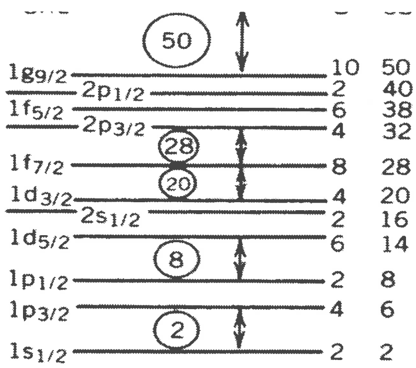
INSTRUCTIONS TO STUDENTS

1. Please write your Student number only. Do not write your name.
2. This assessment paper contains **Four** questions and comprises **Twelve** printed pages.
3. Students are required to answer **ALL** questions.
4. Answers to the questions are to be written on this examination booklet.
5. This is a CLOSED BOOK (with authorized materials) examination.
6. A help sheet (A4-size, two-sided, hand-written) is allowed for the examination.
7. Only non-programmable electronic calculator is allowed for the examination.
8. The Clebsch-Gordan coefficient table is attached at the last page.

Matriculation Number: ______________________________

Question	Marks
1	/30
2	/40
3	/10
4	/20
Total	/100

1. The single-particle shell model predicts the following energy level distribution.



(a) Discuss briefly the physical basis of the single-particle shell model for the nucleus and how this model is able to predict the magic numbers 2, 8, 20, 28, 50, 82, 126, 184.

[12 marks]

(b) Using the shell model, compute the spin and parity assignment of the ground state of the following nuclei.

(i) ${}^{33}_{16}S$ [2 marks]

(ii) ${}^{59}_{29}Cu$ [2 marks]

(iii) ${}^{28}_{11}Na$ [4 marks]

(c) A certain odd-parity, shell model state has total and orbital angular momentum quantum numbers j and l, respectively. If the state can hold up to 8 protons (or 8 neutrons), what are the values of j and l? [10 marks]

2. (a) Describe the physical principle underlying the use of a scintillation detector and a photo-multiplier tube in the detection of a gamma ray, including the various interactions between the gamma ray and the scintillator. [15 marks]

Question 2 (a) answer continues …

(b) The ^{207}Pb nucleus has the following sequence of states beginning with the ground state $\frac{1}{2}^-$ (0 MeV), $\frac{5}{2}^-$ (0.57 MeV), $\frac{3}{2}^-$ (0.90 MeV) and $\frac{13}{2}^+$ (1.63 MeV).

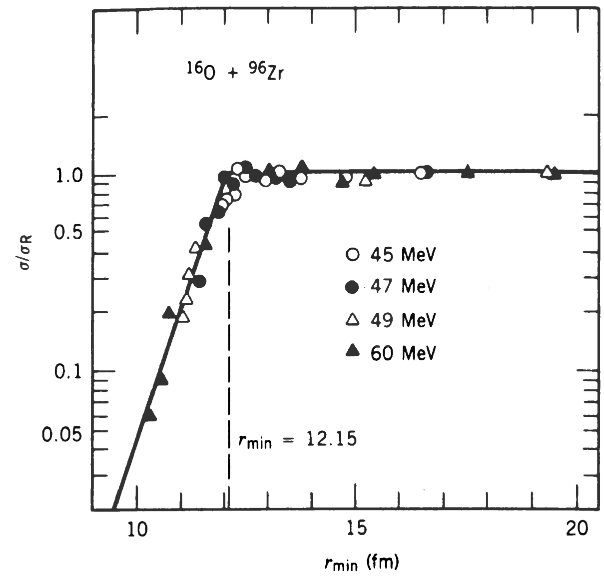
(i) Draw an energy level scheme showing all possible γ transitions and their multipole assignments.

(ii) Indicate the dominant transitions for each excited state. [10 marks]

(c) A nuclear excited state decays by an $E2$ transition to the $\frac{3}{2}^{+}$ ground state. List the possible spin-parity assignments of the excited state. (Hint: $E2$ is one of the dominant transitions.) If there is no evidence of decay by a $M1$ transition, what is the most likely spin-parity assignment of the excited state? [15 marks]

3. The figure below shows the measurements of the elastic scattering cross-section for the scattering of $^{16}_{8}O$ from $^{96}_{40}Zr$. The cross-section is plotted relative to the Rutherford cross-section and the horizontal axis is the distance of closest approach. From the figure, the distance of closest approach in which the Rutherford scattering model breaks down is 12.15 fm. Assuming the nuclei are hard spheres with radius $R = R_0 A^{1/3}$, find the value of R_0. Explain the discrepancy with the value of 1.2 fm from other experiments.

[10 marks]



4. Find the ratios of the cross-sections for the following reactions, assuming the $I = 1$ channel dominates.

Reaction A: $K^- + p \rightarrow \Sigma^0 + \pi^0$

Reaction B: $K^- + p \rightarrow \Sigma^+ + \pi^-$

Reaction C: $\overline{K}^0 + p \rightarrow \Sigma^+ + \pi^0$

Reaction D: $\overline{K}^0 + p \rightarrow \Sigma^0 + \pi^+$

The hadrons above form the following isospin multiplets: $\begin{pmatrix} p \\ n \end{pmatrix}, \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}, \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}, \begin{pmatrix} \overline{K}^0 \\ K^- \end{pmatrix}.$

[20 marks]

Question 4 answer continues …

------------- End of paper -------------

NWK

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2			
m_1	m_2	Coefficients		
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

1/2 × 1/2

m_1	m_2	1
		+1
+1/2	+1/2	1

m_1	m_2	1	0
		0	0
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2

m_1	m_2	1
		-1
-1/2	-1/2	1

1 × 1/2

m_1	m_2	3/2
		+3/2
+1	+1/2	1

m_1	m_2	3/2	1/2
		+1/2	+1/2
+1	-1/2	1/3	2/3
0	+1/2	2/3	-1/3

m_1	m_2	3/2	1/2
		-1/2	-1/2
0	-1/2	2/3	1/3
-1	+1/2	1/3	-2/3

m_1	m_2	3/2
		-3/2
-1	-1/2	1

2 × 1

m_1	m_2	3
		+3
+2	+1	1

m_1	m_2	3	2
		+2	+2
+2	0	1/3	2/3
+1	+1	2/3	-1/3

m_1	m_2	3	2	1
		+1	+1	+1
+2	-1	1/15	1/3	3/5
+1	0	8/15	1/6	-3/10
0	+1	2/5	-1/2	1/10

m_1	m_2	3	2	1
		0	0	0
+1	-1	1/5	1/2	3/10
0	0	3/5	0	-2/5
-1	+1	1/5	-1/2	3/10

m_1	m_2	3	2	1
		-1	-1	-1
0	-1	2/5	1/2	1/10
-1	0	8/15	-1/6	-3/10
-2	+1	1/15	-1/3	3/5

m_1	m_2	3	2
		-2	-2
-1	-1	2/3	1/3
-2	0	1/3	-2/3

m_1	m_2	3
		-3
-2	-1	1

1 × 1

m_1	m_2	2
		+2
+1	+1	1

m_1	m_2	2	1
		+1	+1
+1	0	1/2	1/2
0	+1	1/2	-1/2

m_1	m_2	2	1	0
		0	0	0
+1	-1	1/6	1/2	1/3
0	0	2/3	0	-1/3
-1	+1	1/6	-1/2	1/3

m_1	m_2	2	1
		-1	-1
0	-1	1/2	1/2
-1	0	1/2	-1/2

m_1	m_2	2
		-2
-1	-1	1

2 × 1/2

m_1	m_2	5/2
		+5/2
+2	+1/2	1

m_1	m_2	5/2	3/2
		+3/2	+3/2
+2	-1/2	1/5	4/5
+1	+1/2	4/5	-1/5

m_1	m_2	5/2	3/2
		+1/2	+1/2
+1	-1/2	2/5	3/5
0	+1/2	3/5	-2/5

m_1	m_2	5/2	3/2
		-1/2	-1/2
0	-1/2	3/5	2/5
-1	+1/2	2/5	-3/5

m_1	m_2	5/2	3/2
		-3/2	-3/2
-1	-1/2	4/5	1/5
-2	+1/2	1/5	-4/5

m_1	m_2	5/2
		-5/2
-2	-1/2	1

3/2 × 1/2

m_1	m_2	2
		+2
+3/2	+1/2	1

m_1	m_2	2	1
		+1	+1
+3/2	-1/2	1/4	3/4
+1/2	+1/2	3/4	-1/4

m_1	m_2	2	1
		0	0
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2

m_1	m_2	2	1
		-1	-1
-1/2	-1/2	3/4	1/4
-3/2	+1/2	1/4	-3/4

m_1	m_2	2
		-2
-3/2	-1/2	1

3/2 × 1

m_1	m_2	5/2
		+5/2
+3/2	+1	1

m_1	m_2	5/2	3/2
		+3/2	+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	-2/5

m_1	m_2	5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	-1	1/10	2/5	1/2
+1/2	0	3/5	1/15	-1/3
-1/2	+1	3/10	-8/15	1/6

m_1	m_2	5/2	3/2	1/2
		-1/2	-1/2	-1/2
+1/2	-1	3/10	8/15	1/6
-1/2	0	3/5	-1/15	-1/3
-3/2	+1	1/10	-2/5	1/2

m_1	m_2	5/2	3/2
		-3/2	-3/2
-1/2	-1	3/5	2/5
-3/2	0	2/5	-3/5

m_1	m_2	5/2
		-5/2
-3/2	-1	1

3/2 × 3/2

m_1	m_2	3
		+3
+3/2	+3/2	1

m_1	m_2	3	2
		+2	+2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	-1/2

m_1	m_2	3	2	1
		+1	+1	+1
+3/2	-1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	-2/5
-1/2	+3/2	1/5	-1/2	3/10

m_1	m_2	3	2	1	0
		0	0	0	0
+3/2	-3/2	1/20	1/4	9/20	1/4
+1/2	-1/2	9/20	1/4	-1/20	-1/4
-1/2	+1/2	9/20	-1/4	-1/20	1/4
-3/2	+3/2	1/20	-1/4	9/20	-1/4

m_1	m_2	3	2	1
		-1	-1	-1
+1/2	-3/2	1/5	1/2	3/10
-1/2	-1/2	3/5	0	-2/5
-3/2	+1/2	1/5	-1/2	3/10

m_1	m_2	3	2
		-2	-2
-1/2	-3/2	1/2	1/2
-3/2	-1/2	1/2	-1/2

m_1	m_2	3
		-3
-3/2	-3/2	1

2 × 3/2

m_1	m_2	7/2
		+7/2
+2	+3/2	1

m_1	m_2	7/2	5/2
		+5/2	+5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	-3/7

m_1	m_2	7/2	5/2	3/2
		+3/2	+3/2	+3/2
+2	-1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	-2/5
0	+3/2	2/7	-18/35	1/5

m_1	m_2	7/2	5/2	3/2	1/2
		+1/2	+1/2	+1/2	+1/2
+2	-3/2	1/35	6/35	2/5	2/5
+1	-1/2	12/35	5/14	0	-3/10
0	+1/2	18/35	-3/35	-1/5	1/5
-1	+3/2	4/35	-27/70	2/5	-1/10

m_1	m_2	7/2	5/2	3/2	1/2
		-1/2	-1/2	-1/2	-1/2
+1	-3/2	4/35	27/70	2/5	1/10
0	-1/2	18/35	3/35	-1/5	-1/5
-1	+1/2	12/35	-5/14	0	3/10
-2	+3/2	1/35	-6/35	2/5	-2/5

m_1	m_2	7/2	5/2	3/2
		-3/2	-3/2	-3/2
0	-3/2	2/7	18/35	1/5
-1	-1/2	4/7	-1/35	-2/5
-2	+1/2	1/7	-16/35	2/5

m_1	m_2	7/2	5/2
		-5/2	-5/2
-1	-3/2	4/7	3/7
-2	-1/2	3/7	-4/7

m_1	m_2	7/2
		-7/2
-2	-3/2	1

2 × 2

m_1	m_2	4
		+4
+2	+2	1

m_1	m_2	4	3
		+3	+3
+2	+1	1/2	1/2
+1	+2	1/2	-1/2

m_1	m_2	4	3	2
		+2	+2	+2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	-3/7
0	+2	3/14	-1/2	2/7

m_1	m_2	4	3	2	1
		+1	+1	+1	+1
+2	-1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	-1/14	-3/10
0	+1	3/7	-1/5	-1/14	3/10
-1	+2	1/14	-3/10	3/7	-1/5

m_1	m_2	4	3	2	1	0
		0	0	0	0	0
+2	-2	1/70	1/10	2/7	2/5	1/5
+1	-1	8/35	2/5	1/14	-1/10	-1/5
0	0	18/35	0	-2/7	0	1/5
-1	+1	8/35	-2/5	1/14	1/10	-1/5
-2	+2	1/70	-1/10	2/7	-2/5	1/5

m_1	m_2	4	3	2	1
		-1	-1	-1	-1
+1	-2	1/14	3/10	3/7	1/5
0	-1	3/7	1/5	-1/14	-3/10
-1	0	3/7	-1/5	-1/14	3/10
-2	+1	1/14	-3/10	3/7	-1/5

m_1	m_2	4	3	2
		-2	-2	-2
0	-2	3/14	1/2	2/7
-1	-1	4/7	0	-3/7
-2	0	3/14	-1/2	2/7

m_1	m_2	4	3
		-3	-3
-1	-2	1/2	1/2
-2	-1	1/2	-1/2

m_1	m_2	4
		-4
-2	-2	1

d functions

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 35.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.