Question 1 (a)

$a_1 A$ is the volume term. It reflects the "nearest neighbor" interactions. The binding energy is constant within it's value, so $B_1 \propto R^2 \propto A$.

$a_2 A^{\frac{2}{3}}\left(1+\frac{2}{5}k^2\right)$ is the surface term. The volume term has to subtract this term since the nucleons at the surface feels a smaller nuclear force, as there are lesser neighbours. So $B_2 \propto 4\pi R^2 \propto A^{\frac{2}{3}}$.

$a_3 Z(Z-1)A^{-\frac{1}{3}}\left(1-\frac{1}{5}k^2\right)$ is the Coulomb term, the charge potential. The potential energy of the assumed uniformly charged sphere is $\frac{3}{5}\frac{e^2}{4\pi\varepsilon_0}Z(Z-1)\frac{1}{R}$. It is not Z^2, because the electrostatic repulsion will only exist for more than 2 protons. So $B_3 \propto \frac{Z(Z-1)}{R} \propto Z(Z-1)A^{-\frac{1}{3}}$.

$a_4\frac{(A-2Z)^2}{A}$ is the symmetry term. It accounts for the fact that equal number of protons and neutrons in a nucleus seems to be stable for light nuclei. It is found that too many neutrons in the nucleus reduce the binding energy. However, the effect is small in light nuclei. It can be derived from the Fermi gas model, where Fermi energy is the maximum energy a particle can have in a potential well. This term is the difference between the total energy of the neutrons and protons with the energy for equal number of nucleons.

δ is the pairing term. The binding energy is found to have the properties of $B_{Z,N\text{ even}} > B_{\substack{Z\text{ odd},N\text{ even}\\ N\text{ odd},Z\text{ even}}} > B_{Z,N\text{ odd}}$. So we consider this term as an addition to the binding energy for even-even nuclei, and a subtraction when there are odd-odd nuclei.

Question 1 (b) (i)

For large A, the Coulomb term is very small. a_3 and $A^{-\frac{1}{3}}$ are both very small, and when they multiply each other, that term is negligible. So

$$
\begin{aligned}
a_3 Z(Z-1)\left[(A-1)^{-\frac{1}{3}}-A^{-\frac{1}{3}}\right] &= a_3 Z(Z-1)A^{-\frac{1}{3}}\left[\left(1-\frac{1}{A}\right)^{-\frac{1}{3}}-1\right] \\
&= a_3 Z(Z-1)A^{-\frac{1}{3}}\left(\frac{1}{3A}\right) \\
&= \frac{1}{3}a_3 Z(Z-1)A^{-\frac{4}{3}}
\end{aligned}
$$

Question 1 (b) (ii)

For the 2nd term,

$$a_2\left[(A-1)^{\frac{2}{3}}-A^{\frac{2}{3}}\right]=a_2A^{\frac{2}{3}}\left[\left(1-\frac{1}{A}\right)^{\frac{2}{3}}-1\right]=a_2A^{\frac{2}{3}}\left[1-\frac{2}{3A}-1\right]=-\frac{2}{3}a_2A^{-\frac{1}{3}}$$

For the 4th term,

$$a_4\left[\frac{(A-1-2Z)^2}{A-1}-\frac{(A-2Z)^2}{A}\right]=a_4\left[(A-1)\left(1-\frac{2Z}{A-1}\right)^2-A\left(1-\frac{2Z}{A}\right)^2\right]$$

$$=a_4\left\{(A-1)\left[1-\frac{4Z}{A-1}+\frac{4Z^2}{(A-1)^2}\right]-A\left(1-\frac{4Z}{A}+\frac{4Z^2}{A^2}\right)\right\}$$

$$=a_4\left(A-1-4Z+\frac{4Z^2}{A-1}-A+4Z-\frac{4Z^2}{A}\right)$$

$$=-a^4\left[1-\frac{4Z^2}{A(A-1)}\right]$$

$$\therefore p=\frac{2}{3},\qquad q=4.$$

Question 1 (b) (iii)

The first nuclei has even Z and even N. So $S_{n1}=a_5\left[A^{-\frac{3}{4}}-(A-1)^{-\frac{3}{4}}\right]$.

The second nuclei has odd Z and even N. Since both have the same value of A, then $S_{n2}=-a_5\left[A^{-\frac{3}{4}}-(A-1)^{-\frac{3}{4}}\right]$.

The difference in their neutron separation energies,

$$\Delta S_n=a_5\left[A^{-\frac{3}{4}}-(A-1)^{-\frac{3}{4}}\right]-a_5\left[A^{-\frac{3}{4}}-(A-1)^{-\frac{3}{4}}\right]=2a_5\left[A^{-\frac{3}{4}}-(A-1)^{-\frac{3}{4}}\right]$$

For large nuclei,

$$\Delta S_n=2a_5\left[A^{-\frac{3}{4}}-A^{-\frac{3}{4}}\left(1-\frac{1}{A}\right)^{-\frac{3}{4}}\right]\approx 2a_5A^{-\frac{3}{4}}\left[1-1+\frac{3}{4A}\right]=\frac{3}{2}a_5A^{-\frac{7}{4}}$$

Question 2 (a)

$$\frac{dB}{dk}\geq 0,$$

$$\frac{2}{3}kA^{\frac{2}{3}}\left(a_3\frac{Z^2}{A}-2a_2\right)\geq 0$$

$$a_3\frac{Z^2}{A}\geq 2a_2$$

$$\frac{Z^2}{A}\geq 2\frac{a_2}{a_3}$$

$$\therefore c=2.$$

Question 2 (b) (i)

$${}^{239}_{92}U\left(\frac{5^+}{2}\right) \xrightarrow{\beta^-} {}^{239}_{93}Np\left(\frac{5^+}{2}\right) + e^- + \bar{\nu}_e, \qquad \Delta I = 0, \Delta\pi = \text{no}$$

It is an allowed decay.

$${}^{239}_{93}Np\left(\frac{5^+}{2}\right) \xrightarrow{\beta^-} {}^{239}_{94}Pu\left(\frac{1^+}{2}\right) + e^- + \bar{\nu}_e, \qquad \Delta I = 2, \Delta\pi = \text{no}$$

It is a 2nd forbidden decay.

Question 2 (b) (ii)

${}^{239}_{92}U$ and ${}^{239}_{94}Pu$ have odd number of neutrons, while ${}^{239}_{93}Np$ has odd number of protons.

Question 2 (c) (i)

$$\frac{7^+}{2} \to \frac{11^-}{2} \quad \Delta I = 2, \Delta\pi = \text{yes} \quad \Rightarrow \quad \text{1st forbidden decay}$$

$$\frac{7^+}{2} \to \frac{3^+}{2} \quad \Delta I = 2, \Delta\pi = \text{no} \quad \Rightarrow \quad \text{2nd forbidden decay}$$

Question 2 (c) (ii)

There will be more of the excited state, because 1st forbidden decay means that it has a higher probability to decay compared to 2nd forbidden decay.

Question 2 (c) (iii)

$4 \le I \le 7, \qquad \Delta\pi = \text{yes}$

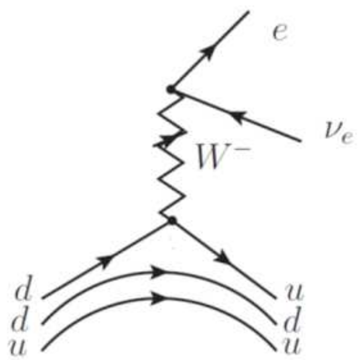
$\therefore M4, E5, M6, E7$

Question 2 (d) (i)

$$\begin{pmatrix} u \\ d \\ d \end{pmatrix} \to \begin{pmatrix} u \\ u \\ d \end{pmatrix}$$

It is through weak interaction, a d quark converts to a u quark.

Question 2 (d) (ii)



Question 3 (a)

Photopeak : γ rays lose all their energies in the detector.

Compton continuum : happens when γ rays undergo Compton scattering and leave before they deposit all their energy.

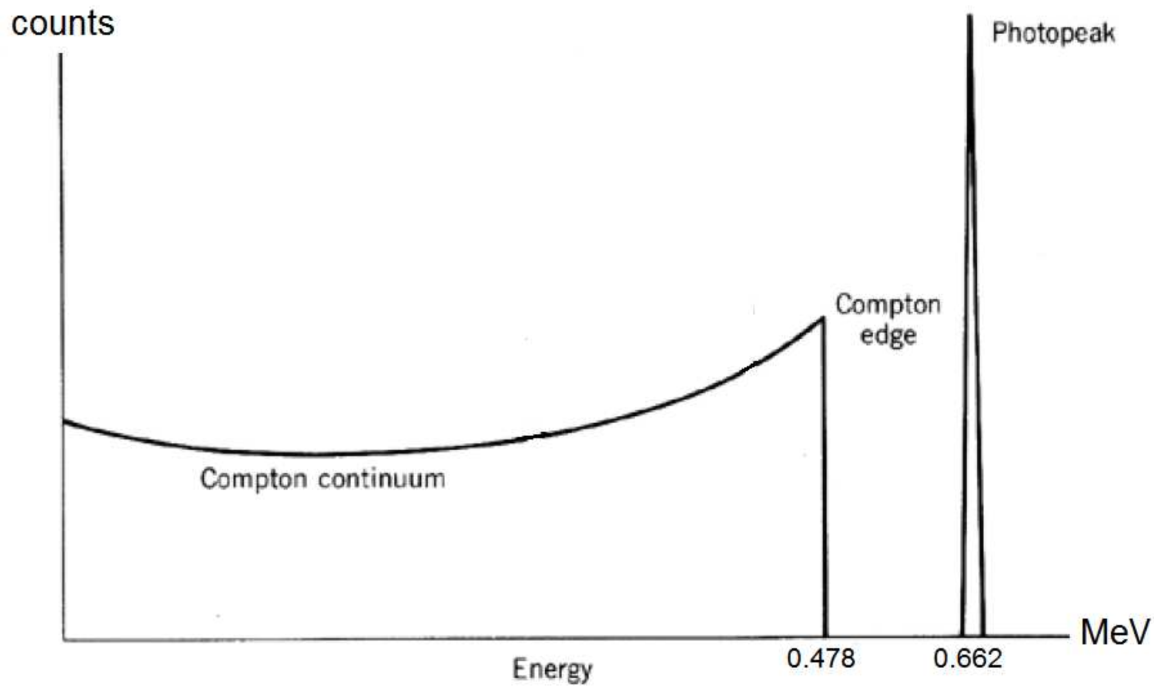
Double escape peak : occurs when a pair-production followed by position annihilation, producing 2 photons and both photons escape.

Single escape peak : same as double escape peak, but only one photon escapes.

Question 3 (b)

There are no escape peaks, since $E_\gamma < 1.022\text{MeV}$.

The Compton edge, $T_e = \dfrac{2E_\gamma^2}{mc^2 + 2E_\gamma} = 0.478\text{MeV}.$



Question 3 (c) (i)

$$\frac{E_\gamma}{\frac{2E_\gamma^2}{mc^2 + 2E_\gamma}} = \frac{5}{4}$$

$$\frac{mc^2 + 2E_\gamma}{E_\gamma} = \frac{5}{2}$$

$$mc^2 + 2E_\gamma = \frac{5}{2}E_\gamma$$

$$mc^2 = E_\gamma\left(\frac{5}{2} - 2\right)$$

$$E_\gamma = 2mc^2$$

Question 3 (c) (ii)

$$\frac{1}{4}mc^2 = \frac{mc^2 E_\gamma}{mc^2 + 2E_\gamma}$$

$$\frac{1}{4}mc^2 + \frac{1}{2}E_\gamma = E_\gamma$$

$$E_\gamma = \frac{1}{2}mc^2$$

Question 3 (d) (i)

First scattering,

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{2E_\gamma}{mc^2}} = \frac{E_\gamma mc^2}{mc^2 + 2E_\gamma}$$

Second scattering,

$$E''_\gamma = \frac{\frac{E_\gamma mc^2}{mc^2 + 2E_\gamma}}{1 + \frac{2}{mc^2}\frac{E_\gamma mc^2}{mc^2 + 2E_\gamma}} = \frac{E_\gamma}{1 + \frac{4E_\gamma}{mc^2}}$$

Question 3 (d) (ii)

$$E_\gamma - E''_\gamma = E_\gamma - \frac{E_\gamma}{1 + \frac{4E_\gamma}{mc^2}} = \frac{4E_\gamma^2}{mc^2 + 4E_\gamma}$$

Question 4 (a) (i)

$$p_{i(\text{lab})} = \begin{pmatrix} E_A + m_B \\ \vec{p}_A \end{pmatrix}, \qquad p_{f(\text{CM})} = \begin{pmatrix} m_C + m_D + m_H \\ 0 \end{pmatrix}$$

$$(E_A + m_B)^2 - p_A^2 = (m_C + m_D + m_H)^2$$

$$E_A^2 + 2E_A m_B + m_B^2 - p_A^2 = (m_C + m_D + m_H)^2$$

$$E_A = \frac{(m_C + m_D + m_H)^2 - m_B^2 - m_A^2}{2m_B}$$

$$T_{th} = E_A - m_A = \frac{(m_C + m_D + m_H)^2 - (m_A + m_B)^2}{2m_B}$$

Question 4 (a) (ii)

$$p + p \to p + p + \text{Higgs}$$

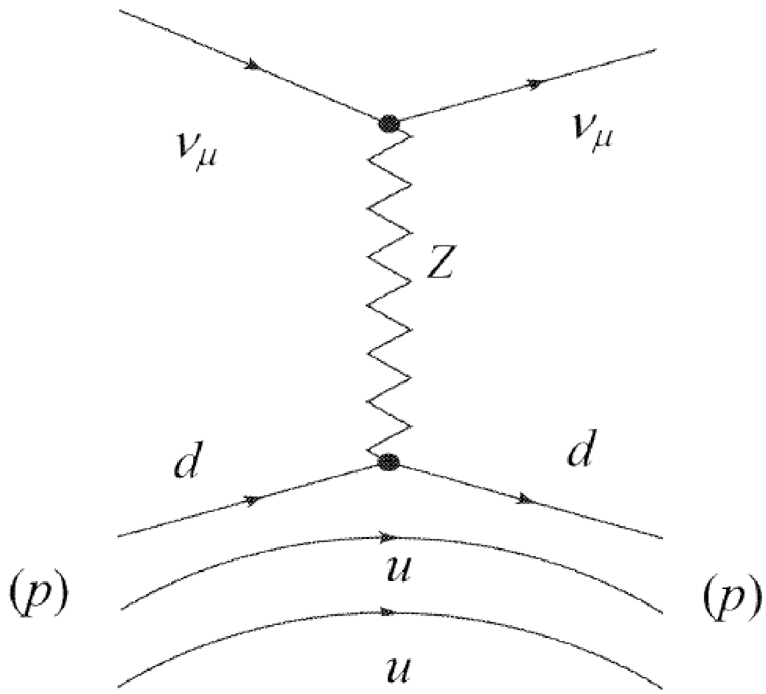
$$T_{th} = \frac{122^2 m_p^2 - 4m_p^2}{2m_p} = 7440m_p = 7440\text{GeV}$$

The threshold energy is freakin' high. The colliding proton needs to be accelerated to such an enormous value of energy, which is quite impossible. If we use colliding beams instead, it will only require $\frac{1}{2}(120)m_p = 60\text{GeV}$ of energy, and so it is better to use colliding beams.

Question 4 (b) (i)

Firstly, the particle ν_e should be ν_μ, because the lepton generation number was violated. Next its corresponding arrow should be pointing outward, as it the lepton number was not conserved.

Question 4 (b) (ii)



Question 4 (c) (i)

Charge conservation means that the initial charge and final charge must be the same. In the given examples, both equations obey this conservation since the charges before and after the reaction are the same.

Charge conjugation is an operation to change the sign of charges by converting its particle to its anti-particle. It changes all internal quantum numbers: charge, baryon number, lepton number and strangeness, while keeping mass, energy, momentum and spin untouched. It is not conserved in weak interaction. The converstion of the first equation to the second equation is charge conjugation.

Question 4 (c) (ii)

Because neutrinos are left-handed.

Question 4 (c) (iii)

We introduce a parity change in the equation.

Question 4 (d)

An anti-baryon has 3 anti quarks. Anti quarks have either charge $-\frac{2}{3}$ or $\frac{1}{3}$. We observe by mixing and matching, there are only 4 possible charges for anti-baryons:

$$-\frac{2}{3}-\frac{2}{3}-\frac{2}{3}=-2$$
$$-\frac{2}{3}-\frac{2}{3}+\frac{1}{3}=-1$$
$$-\frac{2}{3}+\frac{1}{3}+\frac{1}{3}=0$$
$$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

$\therefore\ Q=2$ for an anti-baryon is not allowed.

Solutions provided by:
John Soo (Q1a-bi, 2ciii, 4c-d)
Bong Kok Wei (Q1bii-iii, 2a-cii, 2d, 3, 4a-b)