NATIONAL UNIVERSITY OF SINGAPORE

PC3232 Nuclear and Particle Physics

(Semester I: AY 2011-12)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **4** questions and comprises **6** printed pages.
2. Answer **ALL** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.

1. The semi-empirical mass formula for the binding energy for a deformed nucleus is given by

$$B(A,Z) = a_1 A - a_2 A^{2/3}\left(1+\frac{2}{5}k^2\right) - a_3 Z(Z-1)A^{-1/3}\left(1-\frac{1}{5}k^2\right) - a_4\frac{(A-2Z)^2}{A} + \delta$$

where $\delta = \begin{cases} +a_5 A^{-3/4} & \text{even } Z, \text{even } N \\ 0 & \text{odd } Z, \text{ even } N \text{ or even } Z, \text{ odd } N \\ -a_5 A^{-3/4} & \text{odd } Z, \text{ odd } N \end{cases}$

and k is the deformation parameter .

(a) Discuss briefly the physical basis for the various terms in the above equation.

[12 marks]

(b) The neutron separation energy is given by $S_n = B\left({}^{A}_{Z}X_N\right) - B\left({}^{A-1}_{Z}X_{N-1}\right)$. For spherical nucleus ($k=0$) with large A, the neutron separation energy can be approximated by

$$S_n \approx a_1 - pa_2 A^{-1/3} - a_4\left(1 - \frac{qZ^2}{A(A-1)}\right) + \text{pairing term contribution}$$

(i) Discuss why the Coulomb term is omitted in the approximated S_n. [2 marks]

(ii) Find the values for p and q. [4+4 marks]

(iii) Consider two large nuclei with the same number of nucleons A. The first nucleus has even number of protons Z and even number of neutrons N, while the second nucleus has 1 less proton and 1 more neutron. Considering only the pairing term contribution, find the difference in the neutron separation energies between these two nuclei. [3 marks]

2. Nuclear fission occurs for heavy nuclei where the $Z(Z-1)$ factor in the Coulomb term in the semi-empirical mass formula for the binding energy can be replaced by Z^2.

(a) Using the binding energy equation in question 1, we can show that the limiting condition for spontaneous fission to occur is given by

$$\frac{Z^2}{A} \geq C\frac{a_2}{a_3}.$$

Find C. [4 marks]

(b) Two fissile materials used to sustain chain reaction in nuclear fission are ${}^{235}_{92}U$ and ${}^{239}_{94}Pu$. Only ${}^{235}_{92}U$ is natural occurring, ${}^{239}_{94}Pu$ must be produced artificially through the following reactions

$$ {}^{238}_{92}U + {}^{1}_{0}n \longrightarrow {}^{239}_{92}U + \gamma $$

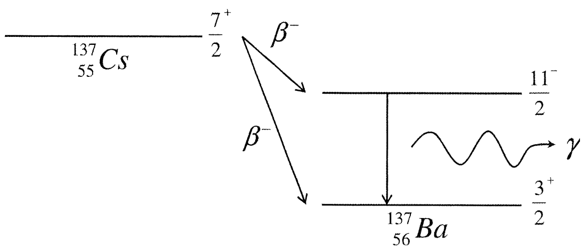
$$ {}^{239}_{92}U\left(\frac{5}{2}^{+}\right) \xrightarrow{\beta^-} {}^{239}_{93}Np\left(\frac{5}{2}^{+}\right) + e^- + \bar{\nu}_e $$

$$ {}^{239}_{93}Np\left(\frac{5}{2}^{+}\right) \xrightarrow{\beta^-} {}^{239}_{94}Pu\left(\frac{1}{2}^{+}\right) + e^- + \bar{\nu}_e $$

(i) Determine the degree of forbiddeness for the 2 beta decays above. [4 marks]

(ii) Give the reason why the total angular momentum of ALL the nuclei involved in the beta decays above are "half-integers". [2 marks]

(c) A typical radioactive product from the fission process is ${}^{137}_{55}Cs$. ${}^{137}_{55}Cs$ can undergo beta decay to either the excited or the ground state of ${}^{137}_{56}Ba$ as shown in the figure below.



(i) Determine the degree of forbiddeness for the 2 beta decays shown in the figure above. [4 marks]

(ii) Given a certain amount of initial ${}^{137}_{55}Cs$, discuss whether you will find more excited ${}^{137}_{56}Ba$ or more ground state ${}^{137}_{56}Ba$ after sometime. [2 marks]

(iii) Determine all possible types of multipole radiations in the gamma decay of the excited ${}^{137}_{56}Ba$ to ground state ${}^{137}_{56}Ba$. [4 marks]

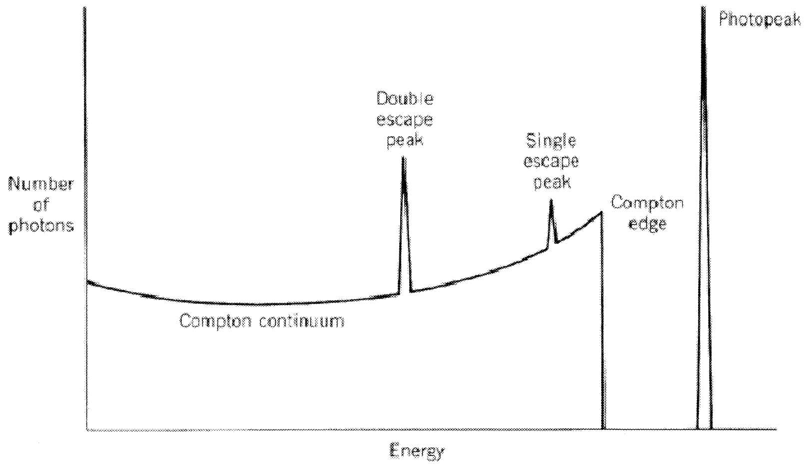
(d) Consider the usual beta decay

$$ n \longrightarrow p + e^- + \bar{\nu} $$

(i) Write down the fundamental decay at the quark level. [2 marks]

(ii) Draw the Feynman diagram, at the quark level, for the above decay. [3 marks]

3. In a Compton scattering event, the energy E_γ' of the scattered photon can be derived to be $E_\gamma' = \dfrac{E_\gamma}{1+\dfrac{E_\gamma}{mc^2}(1-\cos\theta)}$ where θ is the scattering angle of the photon. In the detection of gamma radiations, a typical (toy model) monoenergetic gamma ray spectrum is shown below.



(a) Explain the origin of the photopeak, Compton continuum, double escape peak and single escape peak. [8 marks]

(b) The energy of the photon in the decay of $^{137}_{56}Ba$ is given by 0.662 MeV. Sketch the expected gamma ray spectrum. You are to label the photopeak, Compton edge, single escape peak and double escape peak (if any) by their corresponding energies. [5 marks]

(c) Find the energy E_γ of the monoenergetic photon in terms of electron mass, m if:

(i) the ratio of the energy of Compton edge to the photopeak is given by 4/5. [3 marks]

(ii) the minimum energy carried away by the photon after a single Compton scattering event is given by $\frac{1}{4}mc^2$. [3 marks]

(d) The energy of the monoenergetic photon is E_γ.

(i) Compute the minimum energy carried away by the photon after two Compton scattering events. [4 marks]

(ii) Hence, or otherwise, compute the energy of the Compton edge for this situation. [2 marks]

4. High energy particle collisions can be of the fixed target collisions or the colliding beams.

(a) Suppose we have a collision as follows:

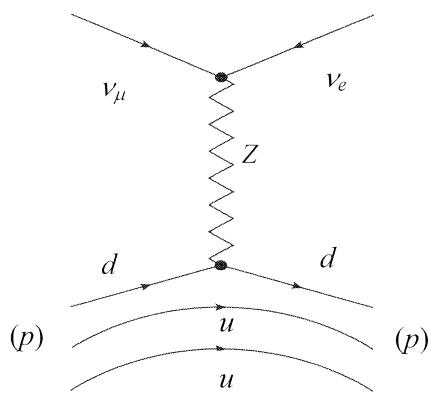
$$A + B \rightarrow C + D + H$$

(i) Show that the threshold energy in the laboratory frame (particle B is at rest) is given by (the speed of light is set to unity, $c = 1$)

$$T_{th}^{lab} = \frac{(m_C + m_D + m_H)^2 - (m_A + m_B)^2}{2m_B}$$ [4 marks]

(ii) If particle H is a Higgs boson (mass of Higgs boson $\sim$ 120 GeV or 120 m_p), use the Higgs boson to illustrate the advantages of using colliding beams over the fixed target collisions, where particle A, B, C, D are all protons. (You may express the energies in term of m_p.) [4 marks]

(b) Consider the following neutral weak interaction process, $p + \nu_\mu \rightarrow p + \nu_\mu$



(i) Identify the mistake(s) in the above Feynman diagram and indicate which conservation law(s) is(are) being violated. [3 marks]

(ii) Draw the correct Feynman diagram for the above process. [2 marks]

(c) Consider the following reactions:

$$p + \pi^+ \rightarrow p + \pi^+ \qquad \bar{p} + \pi^- \rightarrow \bar{p} + \pi^-$$

(i) Explain the difference between charge conservation and charge conjugation conservation. You may use the above reactions for illustrations. [4 marks]

(ii) If we perform charge conjugation operation on the usual beta decay $n \rightarrow p + e^{-} + \overline{\nu}(R)$ where R indicates that the anti-neutrino is right-handed, we get

$$\overline{n} \rightarrow \overline{p} + e^{+} + \nu(R)$$

Charge is conserved for this decay. Give the reason why this decay does not exist in nature. [2 marks]

(iii) What can be done such that the decay of the above anti-neutron can take place? [2 marks]

(d) Discuss why an anti-baryon with electronic charge +2 cannot exist. [4 marks]

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