NATIONAL UNIVERSITY OF SINGAPORE

PC3235 SOLID STATE PHYSICS I

(Semester I: AY 2014-15)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Write your student number only. **Do not write your name.**
2. This examination paper contains **FOUR** questions and comprises **THREE** printed pages.
3. Answer **ANY THREE** questions.
4. All questions carry equal marks.
5. Start each question on a new page.
6. This is a CLOSED BOOK assessment.
7. Programmable calculators are NOT allowed.
8. A book of constants is provided.
9. One Help Sheet (A4 size, both sides, handwritten) is allowed for this examination.

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DEPARTMENT OF PHYSICS

PC3235 – SOLID STATE PHYSICS I

2014/2015

Explain your working clearly. State all principles and assumptions used, and explain all symbols used.

1. (a)

Consider the uniform dilation $\delta = 3e_{xx} = 3e_{yy} = 3e_{zz}$ of a cubic crystal, where $e_{\alpha\beta}$ are the strain components. The elastic energy density is given by

$$U = \tfrac{1}{2}C_{11}(e_{xx}^2 + e_{yy}^2 + e_{zz}^2) + \tfrac{1}{2}C_{44}(e_{yz}^2 + e_{zx}^2 + e_{xy}^2) + C_{12}(e_{yy}e_{zz} + e_{zz}e_{xx} + e_{xx}e_{yy}),$$

where C_{11} , C_{12} and C_{44} are elastic stiffness constants.

(i) If $U = B\delta^2/2$, express the bulk modulus B in terms of the elastic stiffness constants.

(ii) The elastic stiffness constants of a certain cubic crystal are C_{11} = 5.233×10^{11} Pa, $C_{12} = 2.045 \times 10^{11}$ Pa and $C_{44} = 1.607 \times 10^{11}$ Pa. Calculate its bulk modulus.

(b)

The scattering amplitude of x-rays diffracted by a crystal is given by

$$F = \int n(\mathbf{r})\exp[i(\mathbf{k} - \mathbf{k}')\cdot\mathbf{r}]dV,$$

where the integral is over the volume of the crystal, $n(\mathbf{r})$ is the electron number density, and $\mathbf{k}$ and $\mathbf{k}'$ are the respective incident and scattered wavevectors.

(i) Express $n(\mathbf{r})$ as a Fourier series in the reciprocal lattice vectors $\mathbf{G}$. What is the justification for this?

(ii) Hence, show that for diffraction to occur the scattering vector $\Delta\mathbf{k}$ (= $\mathbf{k}' - \mathbf{k}$) must equal $\mathbf{G}$.

(iii) Hence, show that a necessary diffraction condition is $2\mathbf{k}\cdot\mathbf{G} = G^2$.

2. (a) Show that at low temperatures, in the Debye model, the heat capacity C of a d-dimensional dielectric crystal is proportional to T^d, where T is the temperature.

(b) At low temperatures, the measured heat capacity of crystalline potassium is given by $C = 2.08T + 2.57T^3$ (mJ·mol^{-1}·K^{-1}), where T is the temperature.

(i) Determine the Fermi energy (in eV).

(ii) Determine the Debye temperature.

3. (a)

(i) State Mathiessen's law.

(ii) A copper specimen containing 1% impurity atoms has a free mean path for collision with the impurity atoms of 55 nm. Determine the electrical resistivity of the specimen, given that at 300 K, pure copper has a resistivity of 1.7×10^{-8} Ω·m, a free mean path of 40 nm, and the average thermal velocity is 1.2×10^5 m·s^{-1}.

(b)

The thermal conductivity of an insulator crystal, 3 mm in diameter, has a sharp maximum at 30 K. For the crystal, the Debye temperature $\theta = 1100$ K, and the acoustic velocity is 10^4 m·s^{-1}. If at low temperatures, the heat capacity per unit volume is $C_V = 0.1T^3$ J·m^{-3}·K^{-1}, estimate

(i) the maximum value of the thermal conductivity, and

(ii) the thermal conductivity at 75 K.

4. (i) Show that the product of the concentration of electrons in the conduction band and the concentration of holes in the valence band in a semiconductor at temperature T can be expressed as

$$np = 4\left(\frac{k_B T}{2\pi\hbar^2}\right)^3 \left(m_e m_h\right)^{3/2} \exp\left(-\frac{E_g}{k_B T}\right),$$

where the symbols have the usual meaning. $\left[\text{Note: } \int_0^\infty \exp\left(-z^2\right)dz = \sqrt{\pi}/2\right]$

(ii) A semiconductor crystal contains 10^{21} donor atoms/m^3. If the band gap is 1.15 eV at 450 K, and the effective electron and hole masses are each $0.5m$ (m = rest mass of the electron), use the law of mass action to determine the intrinsic concentration of electrons in the conduction band. State any assumptions made.

- End of paper - KMH