# NATIONAL UNIVERSITY OF SINGAPORE

# PC3235 SOLID STATE PHYSICS I

(Semester I: AY 2016-17)

Time Allowed: 2 Hours

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## INSTRUCTIONS TO CANDIDATES

1. Write down your student number only. **Do not write down your name.**
2. This examination paper contains **FOUR** questions and comprises **FOUR** printed pages.
3. Answer **ANY THREE** questions.
4. All questions carry equal marks.
5. Start each question on a new page.
6. This is a CLOSED BOOK assessment.
7. Programmable calculators are NOT allowed.
8. A book of constants is provided.
9. One Help Sheet (A4 size, both sides, handwritten, lettering size no smaller than 12 points) is allowed for this examination.

**Explain your working clearly. State all principles and assumptions used, and explain all symbols used.**

1. Graphene is a two-dimensional honeycomb crystal which is one-atom layer thick, and has a primitive basis of two inequivalent carbon atoms.
   (i) Determine the primitive translation vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ of the crystal in terms of the carbon-carbon bond length $l$.
   (ii) Determine the Bravais lattice type of the crystal.
   (iii) Draw and label a diagram showing the honeycomb lattice of the two inequivalent carbon atoms. Illustrate, in the same diagram, the primitive unit cell, the conventional unit cell and the primitive translation vectors $\mathbf{a}_1$ and $\mathbf{a}_2$ of the Bravais lattice. State the respective dimensions of these two cells.
   (iv) Determine the primitive reciprocal vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ in terms of $l$.
   (v) What is the Bravais lattice type of the reciprocal lattice?
   (vi) Calculate the area of the reciprocal primitive cell, given that the carbon-carbon bond length $l$ is 0.142 nm.
   (vii) How many acoustic and optical branches are there in the phonon dispersion relation $\omega(K)$ of graphene along the $\Gamma - \mathrm{M}$ direction?

2. A crystal of cubic symmetry undergoes a deformation, with $u$, $v$ and $w$ being the respective $x$-, $y$- and $z$- components of the displacement of the deformation.
   (i) Given that

$$\rho \frac{\partial^2 u}{\partial t^2} = C_{11} \frac{\partial e_{xx}}{\partial x} + C_{12}\left(\frac{\partial e_{yy}}{\partial x} + \frac{\partial e_{zz}}{\partial x}\right) + C_{44}\left(\frac{\partial e_{xy}}{\partial y} + \frac{\partial e_{zx}}{\partial z}\right),$$

   show that

$$\rho \frac{\partial^2 u}{\partial t^2} = C_{11} \frac{\partial^2 u}{\partial x^2} + C_{44}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + (C_{12}+C_{44})\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right),$$

   where $\rho$ is the density, $e_{\alpha\beta}$ $(\alpha, \beta = x, y, z)$ are the strain components, and $C_{11}$, $C_{12}$ and $C_{44}$ are the elastic stiffness constants.
   (ii) Hence, determine the dispersion relation $\omega(K)$ for a longitudinal elastic wave propagating in the [111] direction of the crystal.

(iii) Hence, evaluate the velocity of such a longitudinal wave in crystalline silver. Silver, which has an atomic mass of 107.87 u, crystallizes in the cubic close-packed structure, with a lattice constant of 0.41 nm. Its elastic constants are $C_{11} = 1.32 \times 10^{11}$, $C_{12} = 0.97 \times 10^{11}$ and $C_{44} = 0.51 \times 10^{11}$ Pa.

3. (i) Show that the wave equation of an electron in a linear lattice, of lattice constant $a$,

$$\left[p^2/(2m) + U(x)\right]\psi(x) = \varepsilon\,\psi(x),$$

where the eigenfunctions $\psi_k(x) = \sum_k C(k)e^{ikx}$ ($k$ is real), can be written in the form of the central equation

$$(\lambda_k - \varepsilon)\,C(k) + \sum_G U_G C(k-G) = 0,$$

where the kinetic energy of the free electron $\lambda_k = \hbar^2k^2/(2m)$, and $G$ is the reciprocal lattice vector.

(ii) Suppose that only the two Fourier components $U_{G_1}$ and $U_{-G_1}$ of the potential energy $U(x)$ are significant, with $U_{G_1} = U_{-G_1}$ $(= U_1)$, where $G_1 = 2\pi/a$.

(a) Find an expression for the energy $\varepsilon_k$ of an electron in the first (lower) energy band, for $k$ near the first Brillouin zone boundary in terms of $\lambda_k$, $\lambda_{k-G_1}$ and $U_1$.

(b) Hence, calculate the velocity (in m/s) of a nearly free electron in the first energy band of a one-dimensional crystal for $k = \frac{1}{4}G_1$, $a = 0.40$ nm, and $U_1 = 1.6$ eV.

4. (a) Define the effective mass of a charge carrier in a semiconductor crystal. Briefly describe, with the aid of equations, how the effective mass of a charge carrier can be measured using the technique of cyclotron resonance.

(b) The Hall coefficient of an extrinsic semiconductor sample is $+3.85 \times 10^{-4}\ m^3C^{-1}$. If the conductivity of the sample is 108.00 $\Omega^{-1}m^{-1}$, determine (i) the concentration (in $m^{-3}$), and (ii) the mobility of the majority charge carriers. (iii) What is the majority carrier type?

(c) For a certain intrinsic semiconductor, the gradient of the "ln $\sigma$ versus $1/T$" plot is −6530 K, where $\sigma$ is the electrical conductivity and $T$ the temperature. Determine (i) the energy bandgap (in eV), and (ii) the wavelength of the fundamental absorption edge of this semiconductor.

**- End of paper -** KMH