NATIONAL UNIVERSITY OF SINGAPORE

PC3236 – COMPUTATIONAL METHODS IN PHYSICS

(Semester II: AY 2008-09)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains FOUR questions and comprises THREE printed pages.
2. Answer any THREE questions.
3. All questions carry equal marks.
4. Answers to the questions are to be written in the answer books.
5. This is a CLOSED BOOK examination.
6. Programmable calculator is NOT allowed to be used in the examination.
7. A Table of Constants is provided.

1. (a) Evaluate

$$\int_0^2 \frac{\sinh x}{x}\,\mathrm{d}x$$

with Romberg integration. Express your answer accurate to four decimal places.

(b) Use Brent's method to determine the root of

$$f(x) = 3x^3 - 10x^2 + 5x + 5$$

that lies in the interval $(2, 3)$. Express your answer accurate to four decimal places.

(c) The data points

x	1	−1	3	2	4	−2
y	−2	−14	18	1	61	−47

lie on a polynomial. Determine this polynomial.

2. (a) Use Heun's method (or the improved Euler method) to solve the nonlinear differential equation

$$\sqrt{x+1}\,\frac{\mathrm{d}y}{\mathrm{d}x} + e^{-x}y^3 = 0,$$

subject to the initial condition $y(0) = 2$. Use a step size of 0.1 to integrate the differential equation from $x = 0$ to $x = 1$.

(b) Evaluate numerically $\int_{1.2}^{2.2} f(x)\,\mathrm{d}x$, where $f(x)$ is represented by the unevenly spaced data

x	1.0	1.5	1.8	2.4
$f(x)$	6.000	6.875	7.952	12.104

You may find the following information on Gauss-Legendre quadrature useful.

$$\int_{-1}^{1} f(\xi)\,\mathrm{d}\xi \approx \sum_{i=1}^{n} W_i f(\xi_i),$$

where the weights W_i and abscissas ξ_i are given in the following table.

$\pm\xi_i$	W_i	$\pm\xi_i$	W_i
$n = 2$		$n = 5$	
0.577350	1.000000	0.000000	0.568889
$n = 3$		0.538469	0.478629
0.000000	0.888889	0.906180	0.236927
0.774597	0.555556	$n = 6$	
$n = 4$		0.238619	0.467914
0.339981	0.652145	0.661209	0.360762
0.861136	0.347855	0.932470	0.171324

3. (a) Use the finite difference method to solve the wave equation of a vibrating string

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},$$

for $0 \leq x \leq 1$. Assume that the string is initially deformed so that at $t = 0$, we have

$$u(x,0) = \begin{cases} 0, & x = 0, \\ e^{-100(x-0.5)^2}, & 0 < x < 1, \\ 0, & x = 1 \end{cases}$$

and the string is also motionless initially. Both ends of the string are held fixed at all time. Using a grid spacing of 0.1 m, 11 spatial grid points and a time step of 0.1 s, calculate the displacement amplitude of the wave motion at $t = 0.2$ s, i.e. $u(x, 0.2)$.

(b) Use a forward-time centered-space approximation to find the difference equation for the 1-D diffusion equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}.$$

By applying the von Neumann stability analysis, derive the stability criterion relating the time step, grid spacing and α.

4. Apply the shooting method to solve

$$(x-2)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 6\sin(x^2)\frac{\mathrm{d}y}{\mathrm{d}x} + (1-x^2)\cos(x)y = 0$$

subject to the boundary conditions, $y'(0) = -5$, and $y'(1) = 2$. Use the simple Euler method with a step size of 0.1 to find an approximate solution to this boundary-value problem.

LHS

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