This document lacks answers for certain questions. Would you like to help us complete them? If yes, Please send your suggested answers to nus.physoc@gmail.com. Thanks! ☺

Question 1 (a)

Question 1 (b)

Not in syllabus.

Question 2 (a)

$\sin x + 3\cos y - 2 = f_1$

$\cos x - \sin y + 0.2 = f_2$

$f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} \cos x & -3\sin y \\ -\sin x & -\cos y \end{pmatrix}$$

$$\Delta x = -J^{-1}f = -\frac{1}{\cos x \cos y + 3\sin x \sin y}\begin{pmatrix} -\cos y & 3\sin y \\ \sin x & \cos x \end{pmatrix}\begin{pmatrix} \sin x + 3\cos y - 2 \\ \cos x - \sin y + 0.2 \end{pmatrix}$$

$\begin{pmatrix} x \\ y \end{pmatrix}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix}_1 = \begin{pmatrix} x \\ y \end{pmatrix}_0 + \Delta x|_{1,1} = \begin{pmatrix} 0.7909 \\ 1.13841 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix}_2 = \begin{pmatrix} 0.791076 \\ 1.12675 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix}_3 = \begin{pmatrix} 0.791168 \\ 1.12674 \end{pmatrix}$

Question 2 (b)

$$\int_0^2 \int_0^2 \frac{x^2 \sin y}{x+y}\,dx\,dy = \int_{-1}^1 \int_{-1}^1 \frac{x^2 \sin y}{x+y}\,dx\,dy$$

Let $x = 1 + \xi,\ y = 1 + \eta$

$\xi/\eta = \pm\sqrt{\frac{3}{5}}, w = \frac{5}{9};\ \xi/\eta = 0, w = 0$

$$\int_{-1}^{1}\int_{-1}^{1}\frac{x^2\sin y}{x+y}\,dx\,dy = \int_{-1}^{1}\int_{-1}^{1}\frac{(1+\xi)^2\sin(1+\eta)}{2+\xi+\eta}\,d\xi\,d\eta$$

Let $f(\xi,\eta) = \dfrac{(1+\xi)^2\sin(1+\eta)}{2+\xi+\eta}$, we have

$$\int_{-1}^{1}\int_{-1}^{1} f(\xi,\eta)\,d\xi\,d\eta = \int_{-1}^{1}\frac{5}{9}f\left(-\sqrt{\frac{3}{5}},\eta\right)+\frac{8}{9}f(0,\eta)+\frac{5}{9}f\left(\sqrt{\frac{3}{5}},\eta\right)dy$$

$$= \text{an addition of 9 terms ...}$$

$$= 1.47793 \text{ (exact answer: } 1.47791)$$

Question 3 (i)

Question 3 (ii)

Question 3 (iii)

Question 4

Solutions provided by:

A/Prof Paul Lim (Question 2)