NATIONAL UNIVERSITY OF SINGAPORE

PC3236 – COMPUTATIONAL METHODS IN PHYSICS

(Semester II: AY 2010-11)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains FOUR questions and comprises THREE printed pages.
2. Answer ALL questions.
3. All questions carry equal marks.
4. Answers to the questions are to be written in the answer books.
5. This is a CLOSED BOOK examination.
6. Programmable calculator is NOT allowed to be used in the examination.

1. (a) Given the following data points

x	2.6	2.8	3.0	3.2	3.4
$f(x)$	9.06045	7.35366	3.81024	−1.91280	−10.04378

determine $f''(3)$ to a truncation error of $\mathcal{O}(h^4)$.

(b) The equations

$$\tan x - \cos y = 1,$$
$$\cos x + 3 \sin y = 0$$

have a solution in the vicinity of the point ($x = 3, y = 3$). Use two iterations of the Newton-Raphson method to improve on the solution.

2. Five numbers are sampled from a uniform random number generator in the range from 0 to 1: 0.8331, 0.6835, 0.7543, 0.1105, 0.2380. Apply Monte Carlo integration with importance sampling to evaluate the integral

$$I = \int_0^1 \frac{1}{1 + x^{3/2}} \, dx,$$

choosing as the importance function $g(x) = 1 - x/2$. Provide an estimate for the standard deviation of your calculated value.

[Note that $\sigma = \sqrt{\dfrac{\frac{1}{N}\sum_i f(x_i)^2 - \left(\frac{1}{N}\sum_i f(x_i)\right)^2}{N-1}}$]

3. Apply the shooting method to solve

$$\frac{d^2y}{dx^2} + 2000x^3y = 0$$

subject to the boundary conditions, $y(0) = 0$, and $y(0.3) = 1$. Use the modified Euler method with a step size of 0.1 to find an approximate solution to the differential equation.

4. Consider the 2D Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad \text{for} \quad 0 < x < 1 \quad \text{and} \quad 0 < y < 2/3,$$

subject to the following boundary conditions

$$u(x,0) = 0, \qquad u(0,y) = 0,$$

$$u(x,2/3) = 2x, \qquad \frac{\partial u(1,y)}{\partial y} = 3y.$$

Use the finite difference method with a square mesh of width $h = 1/3$ to solve the Laplace equation. Give your answers rounded off to four decimal places.

LHS

– END OF PAPER –