NATIONAL UNIVERSITY OF SINGAPORE

PC3238 – FLUID DYNAMICS

(Semester II: AY 2014–15)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name.**
2. This assessment paper contains FOUR (4) questions and comprises FIVE (5) printed pages, inclusive of this cover page.
3. Students are required to answer any THREE (3) questions.
4. Students should write the answers for each question on a new page.
5. This is a CLOSED BOOK examination.
6. One Help Sheet (A4 size, both sides) is allowed for this examination.

1. Let $f(z)$, where $z = x + iy$, be the complex potential of a 2-D, incompressible, and non-viscous fluid flow on the x-y plane in the absence of boundaries.

   (a) Show that if $f(z)$ has no singularities within the circle $|z| = a$, then the complex potential for the flow generated by introducing the cylinder $|z| = a$ into the flow $f(z)$ is given by

   $$w(z) = f(z) + f^*\left(\frac{a^2}{z}\right)$$

   where the function f^* is obtained by taking the complex conjugate of the function f, but leaving its independent argument unchanged.

   [*For example, if* $f(z) = (U - iV)z$, *then* $f^*(z) = (U + iV)z$ *and* $f^*(a^2/z) = (U + iV)(a^2/z)$].

   (b) Describe the flow field generated by the velocity potential

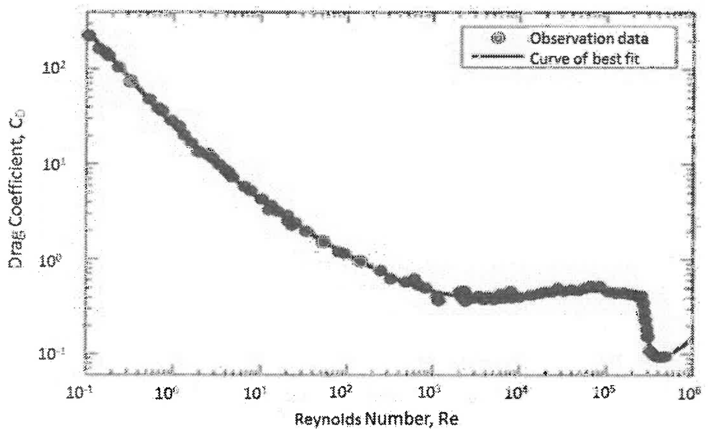
   $$w_\Gamma(z) = \frac{\Gamma}{2\pi i} \ln z.$$

   (c) State the complex potential of the flow generated by a vortex of strength Γ located at $z = b$, where b is a real positive number.

   (d) State the complex potential of the flow generated by the above vortex, but now in the presence of a cylinder $|z| = a$ (with $a < b$) centred at the origin.

   (e) Discuss the motion of the vortex in the presence of the cylinder.

2. The figure below is a log-log plot of Drag Coefficient C_D versus Reynolds Number Re for viscous flow around a sphere.



(a) Consider a viscous fluid of density ρ and viscosity μ flowing around a sphere of radius a. The flow speed far from the sphere is U. Perform a dimensional analysis to show that the drag D experienced by the sphere is expected to depend on the various parameters in the following manner:

$$\frac{D}{\rho U^2 a^2} = \Phi\left(\frac{\rho a U}{\mu}\right),$$

where $\Phi(x)$ is a general function of the argument x.

(b) The Stokes formula gives the drag on a sphere as $D = 6\pi\mu a U$. What is the function $\Phi(x)$ introduced in Part (a) when the Stokes formula is valid?

(c) In which part of the above plot of Drag Coefficient versus Reynolds Number is the Stokes formula applicable, and what should be the gradient of the plot when the Stokes formula is valid?

(d) State the range of values of the Reynolds Number where the drag is approximately proportional to U^2.

(e) Given that $\rho = 1,000\,\mathrm{kg\,m^{-3}}$, $\mu = 0.001\,\mathrm{kg\,m^{-1}s^{-1}}$, $a = 0.02\,\mathrm{m}$, estimate the drag forces for $U = 0.0001\,\mathrm{m\,s^{-1}}, 0.01\,\mathrm{m\,s^{-1}}$, and $20\,\mathrm{m\,s^{-1}}$.

(f) A sharp decrease in the Drag Coefficient is observed at Reynolds Number around $2{\times}10^5$. Briefly describe the physical phenomena that cause the sharp drop in drag.

3. The dispersion relation of 1-D irrotational water waves is given by

$$\omega^2 = \left(gk + \frac{T}{\rho}k^3\right)\tanh(kH).$$

(a) Show that for shallow water gravity waves, the dispersion relation reduces to

$$\omega^2 = gHk^2.$$

(b) What are the phase velocity and group velocity of shallow water gravity waves? Are shallow water gravity waves dispersive?

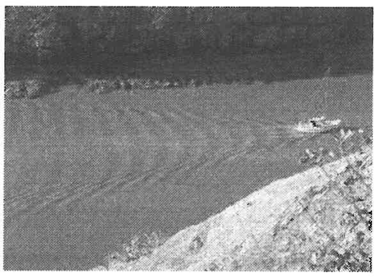
(c) Consider deep water waves with wavelength much smaller than H. Show that the phase velocity has a minimum value of about $0.23\,\mathrm{m\,s^{-1}}$ at wavelength of about 1.7 cm. (*Values of constants:* $g = 9.8\,m\,s^{-2}$, $T = 0.074\,N\,m^{-1}$, $\rho = 1,000\,kg\,m^{-3}$.)

(d) What are the dispersion relations for capillary waves and deep water gravity waves? Determine their phase velocities and group velocities.

(e) An obstacle in a steady stream excites standing waves upstream and downstream of the obstacle. Are the capillary waves found upstream or downstream? Why? Are the gravity waves found upstream or downstream? Why? If the steady stream gains speed, how will it affect the wavelengths of the capillary waves and the gravity waves?

(f) Based on our knowledge of the phase velocity and group velocity of deep water gravity waves, explain why the wave disturbances generated by a ship is confined to a V-shape region of water surface behind the ship, with the ship at the tip of the region. Determine the tip angle of the V-shape region.

(g) If the length of the yacht in the figure below is 10 m, estimate the speed at which it is sailing:



4. In his discussion of turbulent flow development, Landau considered a mode of perturbation with amplitude $A(t) \propto \exp(-i\omega_1 t)\exp(\gamma t)$, with the growth rate γ determined by the Reynolds number Re. There is a critical Reynolds number Re_c such that

$$\gamma(Re) < 0 \quad \text{for} \quad Re < Re_c, \qquad \text{and} \qquad \gamma(Re) > 0 \quad \text{for} \quad Re > Re_c.$$

Since $|A(t)|^2 \propto \exp(2\gamma t)$, we have $\frac{d}{dt}|A|^2 = 2\gamma|A|^2$. However, as the perturbation grows in amplitude, nonlinear effects will come into play to limit its further growth.

(a) Consider first the case when the first nonlinear term $|A|^4$ has a negative coefficient:

$$\frac{d}{dt}|A|^2 = 2\gamma|A|^2 - \alpha|A|^4, \quad \text{where } \alpha \text{ is positive.}$$

i. Show that the amplitute will be limited to the magnitude

$$|A|^2 = 2\gamma/\alpha.$$

ii. Show that when the Reynolds number Re is slightly larger than Re_c, the amplitude increases with the Re as

$$|A| \propto \sqrt{Re - Re_c}.$$

(b) Consider the case when the first nonlinear term has a positive coefficient and the growth of the perturbation is limited by the second nonlinear term:

$$\frac{d}{dt}|A|^2 = 2\gamma|A|^2 + \alpha|A|^4 - \beta|A|^6, \quad \text{where } \alpha \text{ and } \beta \text{ are both positive.}$$

i. Determine the fixed points of the above equation and their stability, for the following three ranges of values of γ:

$$\gamma < -\frac{\alpha^2}{8\beta}, \qquad -\frac{\alpha^2}{8\beta} < \gamma < 0, \qquad \text{and} \qquad 0 < \gamma.$$

ii. Explain why the system is expected to exhibit hysteresis and abrupt changes when the parameter γ is varied up and down through the range of values from $\gamma < -\frac{\alpha^2}{8\beta}$ to $0 < \gamma$.

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