NATIONAL UNIVERSITY OF SINGAPORE

PC3243 Photonics

(Semester II: AY 2012-13)

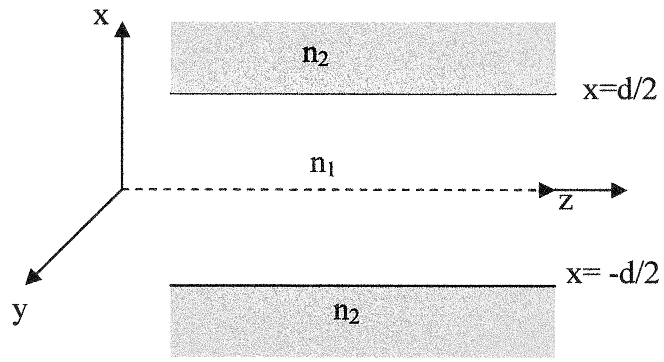
Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **four (4)** questions and comprises **seven (7)** printed pages.
2. Answer **any three (3)** questions.
3. All questions carry equal marks.
4. Answers to the questions are to be written in the answer books.
5. This is a CLOSED BOOK examination.
6. One reference sheet (A4 size, both sides) is allowed for this examination.
7. A Table of Constants is provided

1. <u>Waveguide</u>

Consider a planar waveguide in which the refractive index and dielectric constant varies in the x-direction, as shown below:



For an electromagnetic wave propagating along the z-axis, and assuming that there is no spatial variation of the wave in the y-direction, the Maxwell equations for transverse electric (TE) and transverse magnetic (TM) waves may be reduced respectively to:

$$\frac{\partial^2 E_y}{\partial x^2} + (\omega^2 \varepsilon\mu - \beta^2)E_y = 0$$

$$\frac{\partial^2 H_y}{\partial x^2} + (\omega^2 \varepsilon\mu - \beta^2)H_y = 0$$

(i) By matching the fields and their derivatives according to the boundary conditions, derive the conditions for the allowed guided modes in the form of transcendental equations.

(ii) Explain the graphic approach to solve the transcendental equations for the allowed modes with a sketch, indicating clearly the range of the first even parity mode and that of the first odd parity mode. For sketching take $n_1 = 3.6$ and $n_2 = 3.4$, and the wavelength $\lambda = 1.0$ μm.

(iii) Show that the optical confinement factor Γ is given by

$$\Gamma = \left\{1 + \frac{\cos^2(k_x d/2)}{\gamma\left[d/2 + (1/k_x)\sin(k_x d/2)\cos(k_x d/2)\right]}\right\}^{-1}$$

where

$$k_x = \sqrt{n_r^2 k_0^2 - \beta^2}$$
$$\gamma = \sqrt{\beta^2 - n_r^2 k_0^2}$$

Sketch the optical confinement factor as a function of waveguide thickness d.

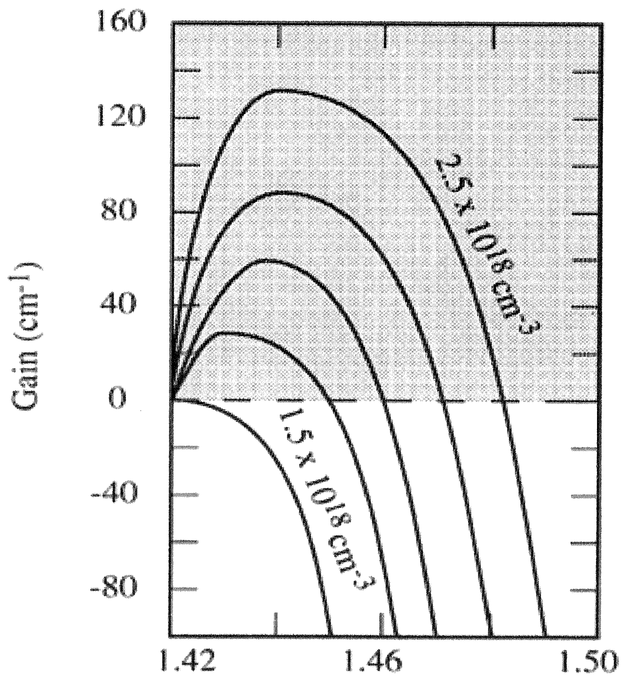
2.

(A) Gain of Semiconductor Laser Materials

(i) Under ideal conditions at $T \approx 0$ K, show that the quasi-Fermi levels are related to the concentrations of the electron-hole pairs injection Δn (= Δp) by

$$E_{fc} - E_{fv} = E_g + (3\pi^2)^{2/3} \frac{\hbar^2}{2m_r} (\Delta n)^{2/3}$$

(ii) Hence explain in what ways the material gain curve (such as shown below for GaAs at $T > 0$ K) depends on the quantity (E_{fc} - E_{fv}).



Note: The material gain $g(\hbar\omega)$ of GaAs (bandgap E_g = 1.42 eV) is given approximately by the equation:

(B) Diode Laser

(i) Describe with diagrams a typical heterostructure diode laser showing the optical cavity, active region, optical confinement of wave profile and other features.

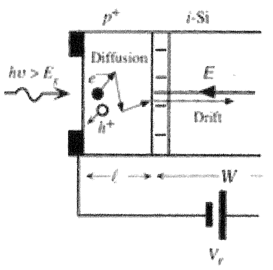
(ii) Explain the conditions for threshold gain $g_{th}(\hbar\omega)$ of a diode laser. Sketch the profile of light emission when the laser is below threshold, at threshold and above threshold. Why is it that the gain of a laser is clamped at $g_{th}(\hbar\omega)$?

(iii) Consider a GaAs/AlGaAs hetero-structure diode laser. The radiative lifetime of the carriers at the threshold is 3.0 ns. If the active region thickness is 0.1 μm, area is $3.0 \times 10^{-5} cm^2$, and the threshold carrier density n_{th} in the active region is $n_{th} = 1.5 \times 10^{18} cm^{-3}$, what is the threshold current I_{th}?

3.

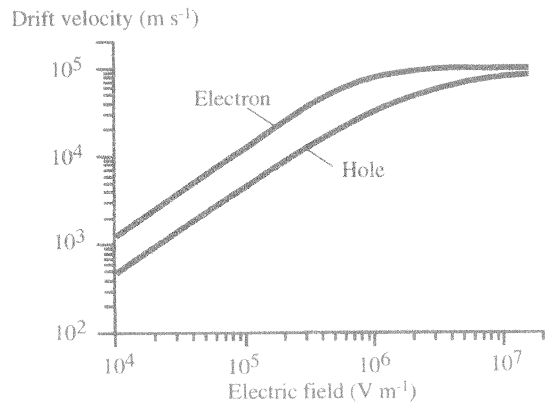
(A) *p-i-n* Photodiode

(i) A reverse biased pin photodiode is illuminated with a short wavelength photon that is absorbed very near the surface as shown.



The photo-generated electron has to diffuse to the depletion region where it is swept into the *i*-layer and drifted across. Assuming that the absorption occurs uniformly very near the surface of the p+ layer, discuss how the speed of response depends on the applied voltage *V* across the photodiode.

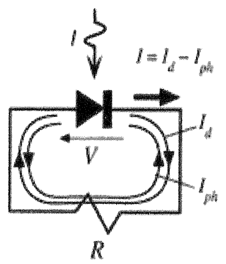
(ii) Given that in time t, on average an electron diffuses a distance l given by $l = [2D_e t]^{1/2}$, what is the speed of response of the device if the *i*-layer is 20 μm, the p^+ layer is 1.0 μm, and the applied voltage is 100 V? The diffusion coefficient D_e of the electrons in the heavily doped p^+ region is approximately $3 \times 10^{-4} m^2 s^{-1}$. The drift velocity vs electric field is shown on the next page.



(B) Solar Cell

(i) Describe with diagrams the principle of operation of an ideal *pn* junction solar cell and the generation of photocurrent I_{ph}.

(ii) Consider the solar cell connected to an external resistive load of resistance R as shown.



Derive an expression for the total current through the solar cell. Sketch the *I-V* characteristics and discuss the direction of currents.

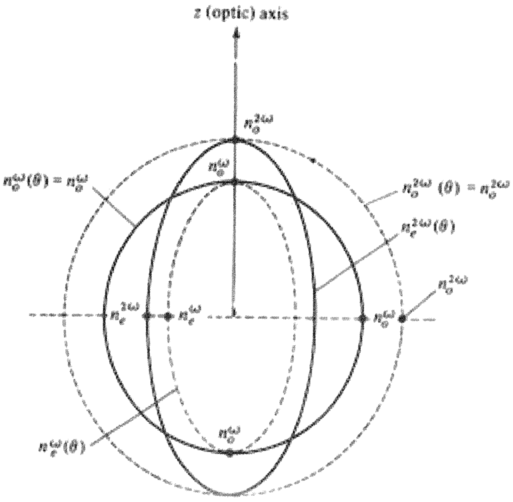
(iii) Consider a Si solar cell with the following parameters:

A = 10 cm X 10 cm, l_n = 0.5μm, W = 2 μm, L_e = 50 μm.

Using a generation rate of $G_0 = 10^{18}$ $cm^{-3}s^{-1}$ at wavelength λ = 1.1 μm and absorption coefficient α = 2000 m^{-1}, determine I_{ph}.

4. Second Harmonic Generation

(i) Explain the requirement for phase matching in second harmonic generation. Illustrate your discussion with a birefringent crystal with refractive index ellipsoid shown on next page.



(ii) Given the index ellipsoid as

$$\frac{1}{\left[n_e^{2\omega}(\theta)\right]^2} = \frac{\cos^2\theta}{\left(n_o^{2\omega}\right)^2} + \frac{\sin^2\theta}{\left(n_e^{2\omega}\right)^2},$$

derive an expression for the phase matching angle θ_m.

(iii) Show that for a small departure from the phase matching angle θ_m such that $\Delta k \neq 0$, the second harmonic power $P_{2\omega}$ after traveling a distance l as a function of θ may be expressed as

$$P_{2\omega}(\theta) \propto \frac{\sin^2[\beta(\theta-\theta_m)]}{[\beta(\theta-\theta_m)]^2}$$

Hence estimate the reduction in efficiency if the angular deviation $(\theta - \theta_m) = 0.001$ rad for a crystal of length $l = 100\ \mu m$ in a crystal of KDP pumped by ruby laser pulse.

Hint: Use Taylor series for the phase mismatch $\Delta k(\theta) = \left[k^{2\omega}(\theta) - 2k^{\omega}(\theta)\right]$.
Show that

$$\Delta k(\theta)l = -\frac{2\omega l}{c}\sin 2\theta_m \frac{\left(n_e^{2\omega}\right)^{-2} - \left(n_0^{2\omega}\right)^{-2}}{2\left(n_0^{\omega}\right)^{-3}}(\theta-\theta_m)$$
$$= 2\beta(\theta-\theta_m)$$

END

Photonics Constants

Properties	Ge	Si	GaAs
Atoms/cm^3	4.42×10^{22}	5.0×10^{22}	4.42×10^{22}
Atomic weight	72.60	28.09	144.63
Breakdown field(V/cm)	$\sim 10^5$	$\sim 3 \times 10^5$	$\sim 4 \times 10^5$
Crystal structure	Diamond	Diamond	Zincblende
Density (g/cm^3)	5.3267	2.328	5.32
Dielectric constant	16.0	11.9	13.1
Effective density of states in conduction band, N_C (cm^{-3})	1.04×10^{19}	2.8×10^{19}	4.7×10^{17}
Effective density of states in valence band, N_V (cm^{-3})	6.0×10^{18}	1.04×10^{19}	7.0×10^{18}
Effective Mass, m^*/m_0			
Electrons	$m_l^* = 1.64$ $m_t^* = 0.082$	$m_l^* = 0.98$ $m_t^* = 0.19$	0.067
Holes	$m_{lh}^* = 0.044$ $m_{hh}^* = 0.28$	$m_{lh}^* = 0.16$ $m_{hh}^* = 0.49$	$m_{lh}^* = 0.082$ $m_{hh}^* = 0.45$
Electron affinity, χ(V)	4.0	4.05	4.07
Energy gap (eV) at 300 K	0.66	1.12	1.424
Intrinsic carrier concentration (cm^{-3})	2.4×10^{13}	1.45×10^{10}	1.79×10^6
Intrinsic Debye length (μm)	0.68	24	2250
Intrinsic resistivity (Ω-cm)	47	2.3×10^5	10^8
Lattice constant (Å)	5.64613	5.43095	5.6533
Minority carrier lifetime (s)	10^{-3}	2.5×10^{-3}	$\sim 10^{-8}$
Mobility (drift) (cm^2/V-s)	3900 1900	1500 450	8500 400

Pockels (r) and Kerr (K) coefficients in various materials.

Material	Crystal	Indices	Pockels Coefficients $\times 10^{-12}$ m/V	K m/V^2	Comment
$LiNbO_3$	Uniaxial	$n_o = 2.272$ $n_e = 2.187$	$r_{13} = 8.6$; $r_{33} = 30.8$ $r_{22} = 3.4$; $r_{51} = 28$		$\lambda = 500$ nm
KDP	Uniaxial	$n_o = 1.512$ $n_e = 1.470$	$r_{41} = 8.8$; $r_{63} = 10.5$		$\lambda \approx 546$ nm
GaAs	Isotropic	$n_o = 3.6$	$r_{41} = 1.5$		$\lambda \approx 546$ nm
Glass	Isotropic	$n_o \approx 1.5$	0	3×10^{-15}	
Nitrobenzene	Isotropic	$n_o \approx 1.5$	0	3×10^{-12}	