NATIONAL UNIVERSITY OF SINGAPORE

PC3246 Nuclear Astrophysics

(Semester II: AY 2013-14)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name.**
2. This examination paper contains **3 questions** and comprises **6 printed pages**.
3. Answer all three questions.
4. Answers to the questions are to be written in the answer books.
5. This is a closed book examination.
6. Data and formulae are included at the end of the paper
7. One A4 page cheat sheet is allowed.

1.

a) Stars that are considerably more massive than the Sun are mainly powered by the CNO cycle List the series of reactions that constitute the CNO cycle. With reference to this list, and the information on reactions given below, explain how and why the CNO cycle tends to increase the abundance of ^{14}N and, to a lesser extent, ^{13}C.

$^{12}C(p,\gamma)$ S(0)=1.5 keV barn
$^{13}C(p,\gamma)$ S(0)=5.5 keV barn
$^{14}N(p,\gamma)$ S(0)=3.3 keV barn
$^{15}N(p,\alpha\gamma)$ S(0)=75 MeV barn

b) Explain why the CNO cycle dominates in massive stars but not in stars of Solar mass and lower. Illustrate your answer with a suitable diagram.

c) After leaving the main sequence, many stars are powered by the triple-alpha process.

$$3\,^4He \rightarrow {}^{12}C$$

This reaction is actually a two-stage process. Write down the two stages separately and calculate their Q-values.

d) One of the reactions in the triple alpha-process is endothermic. What is the source of the additional energy needed? Stating your assumptions, estimate the temperature required for helium fusion to operate from a calculation of the Gamow peak energy.

The masses involved are:
^4He: 4.0026 u
^8Be: 8.0053 u
^{12}C: 12 u (by definition)

2.

a) Briefly explain the significance of the main sequence in the Hertzsprung–Russell diagram.

b) Class A stars are about a factor 10^4 brighter than white dwarfs. The nearest example of an A class star is Sirius A, which is located at a distance of 2.6 parsecs, and has an absolute magnitude of $M = 1.47$. Sirius is orbited by a white dwarf known as Sirius B. What is the apparent magnitude of Sirius B?

c) A binary star has a period of 12 years and the stars orbit with a mutual separation of 7 AU. The stars have apparent bolometric magnitudes of 8.9 and 9.7, respectively. Assuming that they are main sequence stars for which the mass-luminosity relationship $L = \alpha M^4$ holds, determine the mass of each star in Solar masses.

3.

a) A spherical interstellar cloud with a mass of 500 solar masses has a temperature of 30 K and a particle density $n = 10^5\ \text{cm}^{-3}$. Assume the cloud is composed entirely of H. Is this cloud stable against gravitational collapse?

b) Briefly account for the production of each of the following nuclides, in each case explain the principal site of production and the main reactions involved.

$${}^{2}_{1}H$$

$${}^{12}_{6}C$$

$${}^{56}_{26}Fe$$

$${}^{58}_{26}Fe$$

$${}^{60}_{26}Fe$$

PHYSICAL CONSTANTS AND CONVERSION FACTORS

Symbol	Description	Numerical Value
c	velocity of light in vacuum	299 792 458 m s^{-1}, exactly
μ_0	permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}
ε_0	permittivity of vacuum where $c = 1/\sqrt{\varepsilon_0\mu_0}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	gravitational constant	6.673×10^{-11} m^3 kg^{-1} s^{-2}
e	elementary charge	1.602×10^{-19} C
eV	electronvolt	1.602×10^{-19} J
α	fine structure constant, $e^2/4\pi\varepsilon_0\hbar c$	1/137.0
m_e	electron mass	9.109×10^{-31} kg
$m_e c^2$	electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $e\hbar/2m_e$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\alpha^2 m_e c^2/2$	13.61 eV
a_0	Bohr radius, $(1/\alpha)\ (\hbar/m_e c)$	0.5292×10^{-10} m
Å	angstrom	10^{-10} m
m_p	proton mass	1.673×10^{-27} kg
$m_p c^2$	proton rest-mass energy	938.272 MeV
$m_n c^2$	neutron rest-mass energy	939.566 MeV
μ_N	nuclear magneton, $e\hbar/2m_p$	5.051×10^{-27} J T^{-1}
fm	femtometre or fermi	10^{-15} m
b	barn	10^{-28} m^2
u	atomic mass unit, $\frac{1}{12}m(^{12}\mathrm{C}$ atom)	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	triple point temperature	273.16 K
κ	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	molar gas constant, $N_A\kappa$	8.315 J mol^{-1} K^{-1}
σ	Stefan–Boltzmann constant, $(\pi^2/60)(\kappa^4/\hbar^3c^2)$	5.671×10^{-8} W m^{-2} K^{-4}
M_E	mass of earth	5.97×10^{24} kg
R_E	mean radius of earth	6.4×10^6 m
g	standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	standard atmosphere	101 325 Pa, exactly
$M_\odot$	solar mass	1.989×10^{30} kg
$R_\odot$	solar radius	6.960×10^8 m
$L_\odot$	solar luminosity	3.862×10^{26} W
$T_\odot$	solar effective temperature	5800 K
AU	astronomical unit, mean earth–sun distance	1.496×10^{11} m
pc	parsec	3.086×10^{16} m
y	year	3.156×10^7 s

Formulae

Stellar Magnitudes and Distances

$m_1 - m_2 \;=\; -\;2.5\;\log_{10}(f_1 / f_2) \qquad M \;=\; \text{-}2.5\;\log_{10}\;(L / L_{\odot}) + 4.72$

Radiation

$\lambda \nu \;=\; c\;,\; \lambda_{\max} T \;=\; 0.0029\;[K\;m]\;,\; E \;=\; h\nu\;,\; L \;=\; 4\pi R^2 \sigma T^4$

$a = 7.6 \times 10^{-16}\;[\frac{\mathrm{J}}{\mathrm{K}^4 m^3}]$

Jeans density

$$\rho_J > \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\bar{m}}\right)^3$$

Chemical Potential (classical, non-relativistic)

$$\mu(A) = m_A c^2 - kT\;\ln\left[\frac{g_A n_{QA}}{n_A}\right] \qquad n_Q = \left[\frac{2\pi mkT}{h^2}\right]^{3/2}$$

Chemical Potential (classical, relativistic)

$$\mu(A) = -kT \ln \frac{g_A n_Q}{n_A} \qquad n_Q = 8\pi\left[\frac{kT}{hc}\right]^3$$

Gamow Energy

$E_G = 2(\pi\alpha Z_A Z_B)^2 m_r c^2$

Gamow Peak

$$E_0 = \left(\frac{E_G (kT)^2}{4}\right)^{1/3}$$

Kepler III

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} r^3$$

END OF PAPER

[TO]