NATIONAL UNIVERSITY OF SINGAPORE

PC3246 Nuclear Astrophysics

(Semester II: AY 2014-15)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name.**
2. This examination paper contains **3 questions** and comprises **6 printed pages**.
3. Answer all three questions.
4. Answers to the questions are to be written in the answer books.
5. This is a closed book examination.
6. Data and formulae are included at the end of the paper
7. One A4 page cheat sheet is allowed.

1.

a) Briefly discuss the processes that define the minimum and maximum masses for main-sequence stars.

b) We observe a binary system with period 15 yr in which the two stars are 1 and 2 arc sec, respectively, from the center of mass. The system is at a distance of 10 pc. Assuming the system is face-on $(i = 90^{\circ})$ determine the individual masses of the two stars.

c) Jupiter radiates more energy than it receives from the Sun, here we find out if this luminosity may be driven by a Kelvin-Helmholtz contraction.

   i. Jupiter has a luminosity of 8.7×10^{-10} $L_{\odot}$. Jupiter's radius is 7.0×10^{4} km and its mass is 1.9×10^{27} kg. Compute its Kelvin-Helmholtz timescale, assuming a uniform density distribution. Could gravitational contraction power this luminosity for Jupiter's entire lifetime of 4.5 Gyr?

   ii. Use conservation of energy to estimate the rate at which Jupiter's radius is shrinking if Kelvin-Helmholtz contraction drives the luminosity.

2.

a) Assume a star has the same mass and luminosity as the Sun, but consists entirely of He. If the source of energy for that star were the triple alpha process, how long would it take for the star to convert 10 percent of its helium into carbon? Why is this time much less than the main sequence lifetime of the Sun? (The masses needed are given on page 6).

b) After leaving the main sequence, many stars are powered by the triple-alpha process:

$$3\,^{4}He \rightarrow \,^{12}C\,.$$

This reaction is actually a two-stage process. Write down the two stages separately and calculate their individual *Q*-values. What is the significance of the resulting numbers?

c) What is the source of the additional energy needed to supply the energy to allow the triple-alpha process to proceed? Calculate the Gamow peak energy for an estimate of the temperature required for helium fusion to operate.

3.

A stellar atmosphere consists entirely of hydrogen. Assume that the temperature is 2300 K, and that hydrogen molecules (H_2) are in equilibrium with dissociated hydrogen atoms, at a pressure of 100 Pa. Given that the binding energy of the hydrogen molecule is 4.48 eV, estimate the ratio *r* of the densities of H and H_2:

$$r = \frac{n(H)}{n(H_2)}.$$

(Hint: Derive the Saha equation for the hydrogen dissociation reaction, ie:

$$\gamma + H_2 \Leftrightarrow H + H$$

You may assume that the atoms and molecules are in their electronic ground states, so that:

$$g(H) = 4 \text{ and } g(H_2) = 16 .)$$

PHYSICAL CONSTANTS AND CONVERSION FACTORS

Symbol	Description	Numerical Value
c	velocity of light in vacuum	299 792 458 m s^{-1}, exactly
μ_0	permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}
ε_0	permittivity of vacuum where $c = 1/\sqrt{\varepsilon_0\mu_0}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	gravitational constant	6.673×10^{-11} m^3 kg^{-1} s^{-2}
e	elementary charge	1.602×10^{-19} C
eV	electronvolt	1.602×10^{-19} J
α	fine structure constant, $e^2/4\pi\varepsilon_0\hbar c$	1/137.0
m_e	electron mass	9.109×10^{-31} kg
m_ec^2	electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $e\hbar/2m_e$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\alpha^2m_ec^2/2$	13.61 eV
a_0	Bohr radius, $(1/\alpha)$ $(\hbar/m_ec)$	0.5292×10^{-10} m
Å	angstrom	10^{-10} m
m_p	proton mass	1.673×10^{-27} kg
m_pc^2	proton rest-mass energy	938.272 MeV
m_nc^2	neutron rest-mass energy	939.566 MeV
μ_N	nuclear magneton, $e\hbar/2m_p$	5.051×10^{-27} J T^{-1}
fm	femtometre or fermi	10^{-15} m
b	barn	10^{-28} m^2
u	atomic mass unit, $\frac{1}{12}m(^{12}C$ atom)	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	triple point temperature	273.16 K
κ	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	molar gas constant, $N_A\kappa$	8.315 J mol^{-1} K^{-1}
σ	Stefan–Boltzmann constant, $(\pi^2/60)(\kappa^4/\hbar^3c^2)$	5.671×10^{-8} W m^{-2} K^{-4}
M_E	mass of earth	5.97×10^{24} kg
R_E	mean radius of earth	6.4×10^6 m
g	standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	standard atmosphere	101 325 Pa, exactly
$M_\odot$	solar mass	1.989×10^{30} kg
$R_\odot$	solar radius	6.960×10^8 m
$L_\odot$	solar luminosity	3.862×10^{26} W
$T_\odot$	solar effective temperature	5800 K
AU	astronomical unit, mean earth–sun distance	1.496×10^{11} m
pc	parsec	3.086×10^{16} m
y	year	3.156×10^7 s

Formulae

Gravitational energy of a uniform density sphere:

$$E_g = -\frac{3}{5}\frac{G\,M^2}{R}$$

Stellar Magnitudes and Distances

$$m_1 - m_2 = -\,2.5\ \log_{10}(f_1/f_2) \qquad M = -2.5\ \log_{10}\ (L/L_{\odot}) + 4.72$$

Radiation

$$\lambda\nu = c\ ,\ \lambda_{\max}T = 0.0029\ [K\ m]\ ,\ E = h\nu\ ,\ L = 4\pi R^2\sigma T^4$$

$$a = 7.6\times10^{-16}\ [\frac{\mathrm{J}}{\mathrm{K}^4 m^3}]$$

Jeans density

$$\rho_J > \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\bar{m}}\right)^3$$

Chemical Potential (classical, non-relativistic)

$$\mu(A) = m_A c^2 - kT\ \ln\left[\frac{g_A n_{QA}}{n_A}\right] \qquad n_Q = \left[\frac{2\pi mkT}{h^2}\right]^{3/2}$$

Chemical Potential (classical, relativistic)

$$\mu(A) = -kT\ln\frac{g_A n_Q}{n_A} \qquad n_Q = 8\pi\left[\frac{kT}{hc}\right]^3$$

Gamow Energy

$$E_G = 2(\pi\alpha Z_A Z_B)^2 m_r c^2$$

Gamow Peak

$$E_0 = \left(\frac{E_G (kT)^2}{4}\right)^{1/3}$$

Kepler III

$$P^2 = \frac{4\pi^2}{G(m_1+m_2)}r^3$$

The masses involved in the triple alpha process:

^4He: 4.0026 u

^8Be: 8.0053 u

^{12}C: 12 u (by definition)

END OF PAPER

[TO]