NATIONAL UNIVERSITY OF SINGAPORE

PC3246 Nuclear Astrophysics

(Semester II: AY 2015-16)

Time Allowed: 2 Hours

<u>INSTRUCTIONS TO CANDIDATES</u>

1. Please write your student number only. **Do not write your name.**
2. This examination paper contains **3 question**s and comprises **6 printed pages**.
3. Answer all three questions.
4. Answers to the questions are to be written in the answer books.
5. This is a closed book examination.
6. Data and formulae are included at the end of the paper
7. One A4 page cheat sheet is allowed.

1.

An edge-on binary system consists of a neutron star and a main sequence star with $M = 15\ M_{\odot}$ in circular orbits around the center of mass with a period of 20 days. The neutron star has a mass of 2 $M_{\odot}$, a radius of 10 km, and emits blackbody radiation with a surface temperature of 1×10^{6}K.

i. Briefly discuss the Mass-luminosity relation for main-sequence stars.

ii. Calculate the radial velocities of both the main sequence star and the neutron star relative to the center of mass.

iii. The apparent magnitude of the entire binary system (neutron star and main sequence star combined) is +2.8 magnitudes. What is the apparent magnitude of the neutron star?

iv. Estimate the distance to this system (in pc).

2.

a) Briefly answer the following questions:

   i. Why is diffusive energy transport by electrons more efficient than by ions?

   ii. What is meant by bound–free absorption?

b) The approximate temperature -pressure profile in the outer envelope of a white dwarf, as derived from hydrostatic equilibrium and heat flow by radiative diffusion, is given by:

$$\frac{P^2}{2} = \frac{C}{8.5} T^{8.5}. \qquad (1)$$

   i. Show that the neglect of convection is justified, if one assumes an ideal classical gas and the limit for diffusive transport, as derived in class:

$$\frac{dT}{T} < \left(1 - \frac{1}{\gamma}\right) \frac{dP}{P}. \qquad (2)$$

   ii. Use eq. 1 and the hydrostatic equilibrium condition to show that the radiative temperature gradient in the outer envelope of the classical gas surrounding a white dwarf is given by:

$$\frac{dT}{dr} = -\frac{GM\bar{m}}{4.25 r^2 k},$$

   where r is the distance from the center of the white dwarf.

c) Recall that the for an ideal gas the adiabatic index is given by: $\gamma = \frac{2+f}{f}$, where f is the number of degrees of freedom. γ is also ratio of the heat capacities at constant pressure and at constant volume. Show that the critical temperature gradient for the onset of convection:

$$\left.\frac{dT}{dr}\right|_{crit} = \frac{\gamma - 1}{\gamma} \frac{T}{P} \frac{dP}{dr}$$

can also be written as:

$$\left.\frac{dT}{dr}\right|_{crit} = -\frac{g}{c_p},$$

where c_p is the thermal capacity per unit mass at constant pressure and g is the acceleration due to gravity.

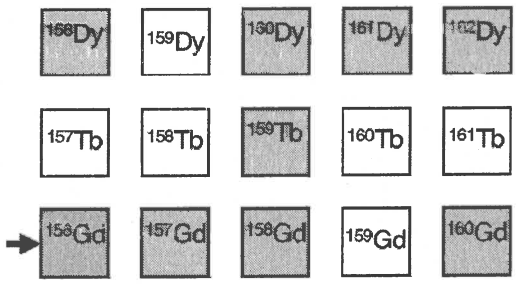
3.

a) Briefly answer the following questions:

i. Which reaction dominates the timescale in the pp chain, and why?

ii. Why is there a high nitrogen abundance during the CNO cycle?

b) The figure below shows a part of the nuclide table, with the elements Gd, Tb and Dy. The shaded squares indicate stable nuclides, and the black arrow shows the isotope supplied by the s-process.



i. Which isotopes are reached by the s process?

ii. Which isotopes are reached by the r process?

iii. Which isotopes are reached by the s process only?

iv. Which isotopes are reached by the r process only?

v. Which isotopes cannot be reached by either process? Suggest a process that might be responsible for the abundance of this isotope?

PHYSICAL CONSTANTS AND CONVERSION FACTORS

Symbol	Description	Numerical Value
c	velocity of light in vacuum	$299\,792\,458$ m s^{-1}, exactly
μ_0	permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}
ε_0	permittivity of vacuum where $c = 1/\sqrt{\varepsilon_0\mu_0}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	gravitational constant	6.673×10^{-11} m^3 kg^{-1} s^{-2}
e	elementary charge	1.602×10^{-19} C
eV	electronvolt	1.602×10^{-19} J
α	fine structure constant, $e^2/4\pi\varepsilon_0\hbar c$	1/137.0
m_e	electron mass	9.109×10^{-31} kg
m_ec^2	electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $e\hbar/2m_e$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\alpha^2 m_ec^2/2$	13.61 eV
a_0	Bohr radius, $(1/\alpha)\ (\hbar/m_ec)$	0.5292×10^{-10} m
Å	angstrom	10^{-10} m
m_p	proton mass	1.673×10^{-27} kg
m_pc^2	proton rest-mass energy	938.272 MeV
m_nc^2	neutron rest-mass energy	939.566 MeV
μ_N	nuclear magneton, $e\hbar/2m_p$	5.051×10^{-27} J T^{-1}
fm	femtometre or fermi	10^{-15} m
b	barn	10^{-28} m^2
u	atomic mass unit, $\frac{1}{12}m(^{12}\text{C atom})$	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	triple point temperature	273.16 K
κ	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	molar gas constant, $N_A\kappa$	8.315 J mol^{-1} K^{-1}
σ	Stefan–Boltzmann constant, $(\pi^2/60)(\kappa^4/\hbar^3c^2)$	5.671×10^{-8} W m^{-2} K^{-4}
M_E	mass of earth	5.97×10^{24} kg
R_E	mean radius of earth	6.4×10^6 m
g	standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	standard atmosphere	101 325 Pa, exactly
$M_\odot$	solar mass	1.989×10^{30} kg
$R_\odot$	solar radius	6.960×10^8 m
$L_\odot$	solar luminosity	3.862×10^{26} W
$T_\odot$	solar effective temperature	5800 K
AU	astronomical unit, mean earth–sun distance	1.496×10^{11} m
pc	parsec	3.086×10^{16} m
y	year	3.156×10^7 s

Formulae

Mass - Luminosity relation for main sequence stars:

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}$$

Gravitational energy of a uniform density sphere:

$$E_g = -\frac{3}{5}\frac{G\,M^2}{R}$$

Stellar Magnitudes and Distances

$$m_1 - m_2 = -2.5\ \log_{10}(f_1/f_2) \qquad M = -2.5\ \log_{10}\ (L/L_\odot) + 4.72$$

Radiation

$$\lambda\nu = c\ ,\ \lambda_{\max}T = 0.0029\ [K\ m]\ ,\ E = h\nu\ ,\ L = 4\pi R^2\sigma T^4$$

Jeans density

$$\rho_J > \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\bar{m}}\right)^3$$

Chemical Potential (classical, non-relativistic)

$$\mu(A) = m_A c^2 - kT\ \ln\left[\frac{g_A n_{QA}}{n_A}\right] \qquad n_Q = \left[\frac{2\pi mkT}{h^2}\right]^{3/2}$$

Chemical Potential (classical, relativistic)

$$\mu(A) = -kT\ln\frac{g_A n_Q}{n_A} \qquad n_Q = 8\pi\left[\frac{kT}{hc}\right]^3$$

Gamow Energy

$$E_G = 2(\pi\alpha Z_A Z_B)^2 m_r c^2$$

Gamow Peak

$$E_0 = \left(\frac{E_G(kT)^2}{4}\right)^{1/3}$$

Kepler III

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)}r^3$$

END OF PAPER

[TO]