**Question 1 (a)**
From the diagram,
$P = 3.5$ days
$v_{1r} = 120 \text{ km s}^{-1}$
$v_{2r} = 50 \text{ km s}^{-1}$
$i = 90°$

Kepler's 3rd Law states that
$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)}(r_1 + r_2)^3 \quad \Rightarrow \quad m_1 + m_2 = \frac{4\pi^2}{GP^2}(r_1 + r_2)^3$$

We also have
$$v = \frac{2\pi r}{P} \quad \Rightarrow \quad r_1 = \frac{v_1 P}{2\pi}, \qquad r_2, = \frac{v_2 P}{2\pi}$$

So our previous equation now becomes
$$m_1 + m_2 = \frac{P}{2\pi G}(v_1 + v_2)^3 = \frac{P}{2\pi G}\frac{(v_{1r} + v_{2r})^3}{\sin^3 i} = \frac{P}{2\pi G}(v_{1r} + v_{2r})^3$$

At the centre of mass, $m_1 r_1 = m_2 r_2$, so
$$\frac{m_1}{m_2} = \frac{r_2}{r_1} = \frac{v_2}{v_1} = \frac{\frac{v_{2r}}{\sin i}}{\frac{v_{1r}}{\sin i}} = \frac{v_{2r}}{v_{1r}} \quad \Rightarrow \quad m_2 = \frac{v_{1r}}{v_{2r}} m_1$$

So with the help of the equation above,
$$m_1 + \frac{v_{1r}}{v_{2r}} m_1 = \frac{P}{2\pi G}(v_{1r} + v_{2r})^3$$
$$\therefore m_1 = \frac{P}{2\pi G}\frac{(v_{1r} + v_{2r})^3}{1 + \frac{v_{1r}}{v_{2r}}}, \qquad m_2 = \frac{P}{2\pi G}\frac{(v_{1r} + v_{2r})^3}{1 + \frac{v_{2r}}{v_{1r}}}$$

**Question 1(b)**
$M = M_\odot$
$L = L_\odot$
$X_4 = 1$
$F = 10\%$
$3\,{}^4He \rightarrow {}^{12}C$

Using the nuclear timescale,
$$\tau_N = \frac{F X_4 M Q_m}{L} = \frac{0.1 M_\odot}{L_\odot} Q_m = \frac{0.1 M_\odot}{L_\odot}\frac{(3m_{He} - m_C)c^2}{3m_{He}}$$

The sun was assumed to be formed with 71% hydrogen and 27% helium and 2% other heavy elements. The sun primarily burns with the p-p chain which involves a weak process, $4p \rightarrow {}^4He + 2e^+ + 2\nu_e$, which takes a much longer time to burn.

Note: $F$ is the fraction of fuel burned; $X$ (in general) is the percentage of the fuel gas in the star (in this case, it happened to be 100% helium); $Q_m$ is the Q-value of the reaction per unit fuel mass.

**Question 2 (a)**

Primordial nucleosynthesis is a process of nuclear reactions right after the ratio of n-p is frozen at 1/7. The nuclear reaction sequences are

$n + p \to d + \gamma$

$n + d \to {}^3H + \gamma \quad \& \quad p + d \to {}^3He + \gamma$

$p + {}^3H \to {}^4He + \gamma \quad \& \quad {}^3He + n \to {}^4He + \gamma$

The reaction stops here, as there are no stable mass 5 nuclei. The big bang nucleosynthesis led to a universe of 25% helium and 75% hydrogen in mass.

Effects of rapidly decreasing temperature:

At high temperatures, any d formed will be quickly disrupted by collision. Then $n \leftrightarrow p$, they were continually transformed into one another. The n-p ratio decreases rapidly due to the Boltzmann factor, $\frac{N_n}{N_p} = e^{-\frac{\Delta mc^2}{kT}}$, it froze out to 1/5 and continues to decrease as neutrons are not stable ($\beta$ decay into protons).

**Question 2 (b) (i)**

$$\frac{n_n}{n_p} = \frac{1}{5} = e^{-\frac{(m_n - m_p)c^2}{kT}} \quad \Rightarrow \quad T = \frac{(m_n - m_p)c^2}{k \ln 5}$$

**Question 2 (b) (ii)**

Because the process $n + p \leftrightarrow d + \gamma$ was a two-way process: at that time the temperature was still very high, thus photons with $E_\gamma > 2.2\text{MeV}$ (binding energy of d) can break up deuterons.

**Question 2 (b) (iii)**

$$\frac{1}{5} \to \frac{1}{7} \quad \Rightarrow \quad \frac{7}{35} \to \frac{5}{35}$$

Throughout the process, there was a loss of 2/7 of the neutrons. So

$$890s \times \frac{2}{7} = 254s$$

Note: exact answer is 278s.

**Question 2 (b) (iv)**

Helium-4: 2 neutrons + 2 protons. If all neutrons become helium-4, then $n_{he} = \frac{n_n}{2}$. So

$n_n : n_p = 1:7$

$n_n : n_n + n_p = 1:8$

After helium formation,

$$n_{He} : n_p = \frac{1}{2} : 6 = 1:12$$

**Question 3 (a)**
The two main nuclear processes are the s-process and the r-process.

Neutron capture:
- During stellar evolution later stages
- Neutrons released by nuclear collision and photodisintegration
- Neutrons captured by nucleus, neutron-rich isotopes decay via beta decay.

s-process is a slow process, it is responsible for the formation of elements from deuterium to bismuth.

r-process is a fast process, it is able to produce elements from bismuth onwards. It happens during the final evolution states of massive stars, when the iron core collapses and ejects its outer layer to form a supernova. The nuclei capture many neutrons before beta decay effective.

Note: cosmic ray collisions also produce heavy elements in small quantities.

**Question 3 (b)**
$$\frac{N_{235}}{N_{238}} = 0.00723, \qquad \left.\frac{N_{235}}{N_{238}}\right|_{\mathrm{r-p}} = 1.6$$
$$t_{\frac{1}{2},{}^{235}U} = 7.13 \times 10^{8} \text{ years}$$
$$t_{\frac{1}{2},{}^{238}U} = 4.51 \times 10^{9} \text{ years}$$

$$N_{235} = N_{235,0}e^{-\lambda_{235}t}$$
$$N_{238} = N_{238,0}e^{-\lambda_{238}t}$$

Divide the two equations,
$$0.00723 = 1.6e^{-(\lambda_{235}-\lambda_{238})t}$$

$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$, so
$$t = -\frac{\ln \frac{0.00723}{1.6}}{\ln 2}\left(t_{\frac{1}{2},{}^{235}U} - t_{\frac{1}{2},{}^{238}U}\right)$$

**Question 4 (a) (i)**
**Question 4 (a) (ii)**
**Question 4 (a) (iii)**

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Solutions provided by:
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