NATIONAL UNIVERSITY OF SINGAPORE

PC3246 Nuclear Astrophysics

(Semester II: AY 20010-11)

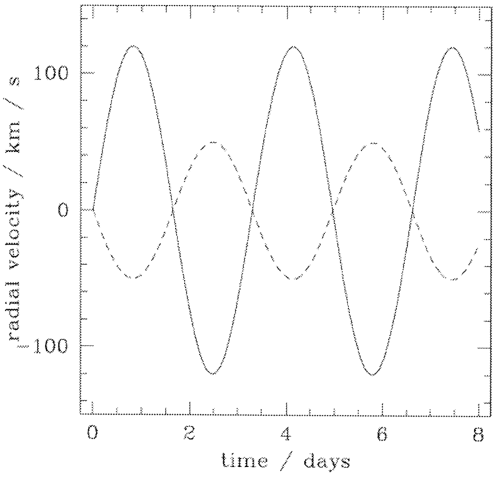
Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains 4 questions and comprises 7 printed pages.
2. Answer any three questions.
3. Answers to the questions are to be written in the answer books.
4. This is a closed book examination.
5. Data and formulae are included at the end of the paper
6. A 1 page cheat sheet is allowed.

1.

a) The radial velocities of two stars in a binary system are measured to have the following variation with time:



Assuming that the system is an eclipsing binary, so that $i \approx 90°$, calculate the masses of the two stars.

b) Assume a star has the same mass and luminosity as the Sun, but consists entirely of He. If the source of energy for that star were the triple alpha nuclear reaction, converting helium into carbon, estimate the time (in years) it would take for the star to convert 10 percent of all of its helium into carbon (See the table at the end of this paper for the masses). Why is this lifetime much less than the main sequence lifetime of the Sun?

2.

a) Describe the production of elements via primordial nucleosynthesis in the early universe, shortly after the Big Bang. Include a brief discussion of the key processes taking place and the effect of the rapidly decreasing temperature.

b) In the early universe the neutron (n) to proton (p) ratio was "frozen out" at a value of 0.2. At a time *t* seconds later, the universe having further cooled, the formation of deuterons started. At the time when deuteron production started the neutron to proton ratio had fallen to 0.135.

   i) Estimate the temperature at which the n:p ratio was frozen at 0.2.

   ii) Why did deuteron production not start immediately at the time when the n:p ratio was frozen to 0.2?

   iii) Given that the lifetime of the neutron is 890 seconds, calculate the time between freeze-out and the start of deuteron production.

   iv) Use the n:p ratio above to estimate the primordial abundance of Helium-4.

3)

a) Outline the two main nuclear processes thought to be responsible for the production of elements heavier than iron. In your discussion indicate the typical timescales relevant to the two processes.

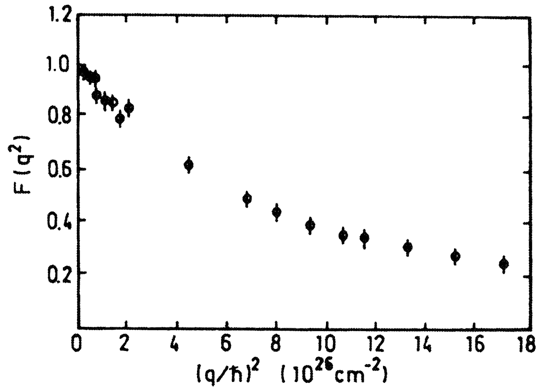
b) The present day value of the uranium isotopic ratio $^{235}U/^{238}U$ is 0.00723. The value of this ratio produced in the r-process is estimated to be about 1.6. Assuming a single supernova was the source of the heavy elements of the Solar System, how long ago would this event have been? The half life of the isotope ^{235}U is 7.13×10^8 years and that of ^{238}U is 4.51×10^9 years.

4.

a) A spherically symmetric nucleus has a radial charge density $\rho(r)$ which is normalized so that $\int_V \rho(r)dv = 1$. The scattering amplitude for coulomb potentials is found in the formulae section at the end of the paper.

i) Show that the form factor for small momentum transfer can be used to estimate the mean square radius from:

$$F(q^2) = 1 - \frac{1}{6\hbar^2} q^2 \langle r^2 \rangle$$



i) The figure above shows experimental results pertaining to the form factor for the proton. Use the result from (ii) to evaluate the root-mean-square (charge) radius of the proton on the basis of the data shown.

ii) Show that the cross section in the Born approximation for high energy electron scattering can be written as the product of a Rutherford term and the electric form factor.

PHYSICAL CONSTANTS AND CONVERSION FACTORS

Symbol	Description	Numerical Value
c	velocity of light in vacuum	299 792 458 m s^{-1}, exactly
μ_0	permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}
ε_0	permittivity of vacuum where $c = 1/\sqrt{\varepsilon_0 \mu_0}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	gravitational constant	6.673×10^{-11} m^3 kg^{-1} s^{-2}
e	elementary charge	1.602×10^{-19} C
eV	electronvolt	1.602×10^{-19} J
α	fine structure constant, $e^2/4\pi\varepsilon_0\hbar c$	1/137.0
m_e	electron mass	9.109×10^{-31} kg
$m_e c^2$	electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $e\hbar/2m_e$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\alpha^2 m_e c^2/2$	13.61 eV
a_0	Bohr radius, $(1/\alpha)$ $(\hbar/m_e c)$	0.5292×10^{-10} m
Å	angstrom	10^{-10} m
m_p	proton mass	1.673×10^{-27} kg
$m_p c^2$	proton rest-mass energy	938.272 MeV
$m_n c^2$	neutron rest-mass energy	939.566 MeV
μ_N	nuclear magneton, $e\hbar/2m_p$	5.051×10^{-27} J T^{-1}
fm	femtometre or fermi	10^{-15} m
b	barn	10^{-28} m^2
u	atomic mass unit, $\frac{1}{12}m(^{12}\mathrm{C}$ atom)	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	triple point temperature	273.16 K
κ	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	molar gas constant, $N_A \kappa$	8.315 J mol^{-1} K^{-1}
σ	Stefan–Boltzmann constant, $(\pi^2/60)(\kappa^4/\hbar^3 c^2)$	5.671×10^{-8} W m^{-2} K^{-4}
M_E	mass of earth	5.97×10^{24} kg
R_E	mean radius of earth	6.4×10^6 m
g	standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	standard atmosphere	101 325 Pa, exactly
$M_\odot$	solar mass	1.989×10^{30} kg
$R_\odot$	solar radius	6.960×10^8 m
$L_\odot$	solar luminosity	3.862×10^{26} W
$T_\odot$	solar effective temperature	5800 K
AU	astronomical unit, mean earth–sun distance	1.496×10^{11} m
pc	parsec	3.086×10^{16} m
y	year	3.156×10^7 s

Formulae

Stellar Magnitudes and Distances

$m_1 - m_2 = -2.5 \log_{10}(f_1 / f_2) \qquad M = -2.5 \log_{10}(L / L_{\odot}) + 4.72$

Radiation

$\lambda \nu = c \,,\; \lambda_{\max} T = 0.0029\, mK \,,\; E = h\nu \,,\; L = 4\pi R^2 \sigma T^4$

Jeans density

$$\rho_J > \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\bar{m}}\right)^3$$

Chemical Potential (classical, non-relativistic)

$$\mu(A) = m_A c^2 - kT \ln\left[\frac{g_A n_{QA}}{n_A}\right] \qquad n_Q = \left[\frac{2\pi mkT}{h^2}\right]^{3/2}$$

Chemical Potential (classical, relativistic)

$$\mu(A) = -kT \ln \frac{g_A n_Q}{n_A} \qquad n_Q = 8\pi\left[\frac{kT}{hc}\right]^3$$

Born approximation scattering amplitude for Coulomb potential

$$f(\theta) = \int_0^\infty \rho(r)\, \frac{\sin(qr/\hbar)}{(qr/\hbar)} 4\pi r^2 dr \quad \times \frac{2mkZe^2}{\hbar q} \int_0^\infty \sin(qs/\hbar) ds$$

Keplers third law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

$m_{He} = 4.0026\ u$

$m_C = 12.000\ u$

END OF PAPER

[T.O.]