Question 1 (a)

For $2^4He \rightarrow {}^8Be, Q = (2m_{He} - m_{Be})c^2$.

For $3^4He \rightarrow {}^{12}C, Q = (3m_{He} - m_C)c^2$.

The different signs show whether a reaction is feasible / spontaneous. If it is a positive sign, it means that the reaction will happen, but if it is a negative sign, it means the reverse reaction will happen.

Question 1 (b)

$e^{-\frac{E}{kT}-\sqrt{\frac{E_G}{E}}}$

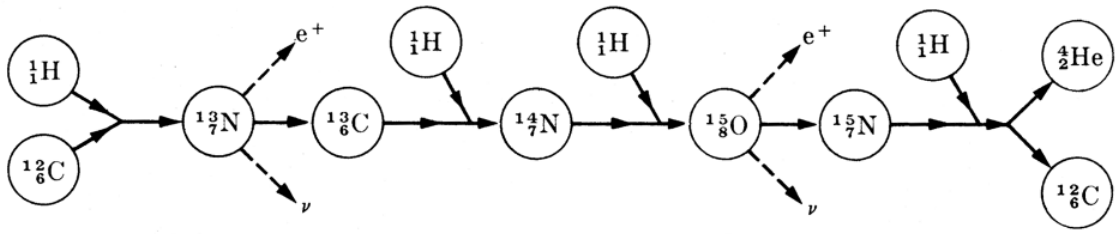
To reeact at energy E, the nuclei need to borrow an energy E from the thermal environment, and the probability of a successful loan is proportional to the Boltzmann factor, $e^{-\frac{E}{kT}}$. To fuse, the nuclei must first penetrate the Coulomb barrier keeping them apart, and the probability of penetration is given by the factor $e^{-\sqrt{\frac{E_G}{E}}}$. The product of these 2 factors indicate that fusion mostly occurs at an energy window of $E_0 = \pm\frac{\Delta}{2}$.

$$\frac{d}{dE}e^{-\frac{E}{kT}-\sqrt{\frac{E_G}{E}}} = 0$$

$$-\frac{1}{kT}+\frac{1}{2}\sqrt{\frac{E_G}{E_0^3}} = 0 \quad \Rightarrow \quad T = \frac{2}{k}\sqrt{\frac{E_0^3}{E_G}}$$

Question 1 (c)

The T^4 dependence of the p-p chain is unable to explain the high luminosities of massive stars. The CNO cycle is a more temperature dependent mechanism for hydrogen burning, and it involves heavier elements with higher Coulomb barriers.



The net result,

$4p \rightarrow {}^4He + 2e^+ + 2\nu_e$

Question 1 (d)

$R = 1.2 \times 10^{-15} A^{\frac{1}{3}}$ [m]

For the reaction $p + {}^{13}C \rightarrow {}^{14}N + \gamma$,

$$V_{max} = \frac{zZe^2}{4\pi\epsilon_0 r_1}, \qquad z = 1, Z = 6$$

r_1 is the sum of the radii of the two molecules. So

$$r_1 = 1.2 \times 10^{-15}\left(13^{\frac{1}{3}} + 1\right)$$

To find the temperature,

$$\frac{3}{2}kT = V_{max}$$

$$\therefore T = \frac{2V_{max}}{3k} = \frac{2}{3k}\frac{6e^2}{4\pi\epsilon_0 \times 1.2 \times 10^{-15}\left(13^{\frac{1}{3}} + 1\right)}$$

Question 2 (a)

$T_E = 18000K$

$R = 2 \times 10^9\text{m}$

$d = 50pc$

i) $L = 4\pi R^2 \sigma T_E^4$

ii) $f = \frac{L}{4\pi d^2}$

iii) $M_B = -2.5\lg\left(\frac{L}{L_\odot}\right) + 4.72$

iv) $m = -2.5\lg\left(\frac{10\text{pc}}{50\text{pc}}\right)^2 + M_B$

Question 2 (b)

$M = 100M_\odot$

$T = 20\text{K}$

$n = 10^5\text{cm}^{-3}$

$X_1 = 1$

$\overline{m} = m_p$ (since it is a cloud of gas at 20K, it shouldn't be ionized)

To know whether the cloud collapses, we compare $\rho = \overline{m}n$ with the Jeans density,

$$\rho_J = \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\overline{m}}\right)^3$$

If $\rho > \rho_J$, then the cloud collapses, and vice versa.

Question 2 (c)

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

$$\int_0^R 4\pi r^3 \frac{dP}{dr}\,dr = -\int_0^R \frac{Gm(r)\rho(r)}{r} 4\pi r^2\,dr$$

$$[P(r)4\pi r^3]_0^R - \int_0^R P(r)12\pi r^2\,dr = -\int_0^M \frac{Gm(r)}{r}\,dm$$

$$3\int_0^R P(r)4\pi r^2\,dr = -\frac{GM^2}{2R}$$

$$3\langle P\rangle V = -E_{GR}$$

$$\langle P\rangle = -\frac{E_{GR}}{3V}$$

$P_c > \langle P\rangle$, so

$$P_c > -\frac{E_{GR}}{3V}$$

$$P_c > -\frac{GM^2}{3VR}$$

$$\therefore P_c > -\frac{GM^2}{8\pi R^4}$$

$$E_{GR} = -2E_{KE}$$

$$-\frac{GM^2}{2R} = -2\frac{3}{2}\frac{M}{\overline{m}}kT$$

$$T = \frac{GM\overline{m}}{6kR}$$

$$\langle T\rangle > T$$

$$\therefore \langle T\rangle > \frac{GM\overline{m}}{6kR}$$

Question 3 (a)

Using $F = ma$,

$$-m\frac{d^2r}{dt^2} = g\Delta M + (P + \Delta P)\Delta A - P\Delta A$$

$$= g\rho(r)\Delta r\Delta A + \Delta P\Delta A$$

$$= g\rho(r)\Delta r\Delta A + \frac{dP}{dr}\Delta r\Delta A$$

$$-\rho(r)\frac{d^2r}{dt^2} = g\rho(r) + \frac{dP}{dr}$$

The inward force balances the outward force. So at $\frac{d^2r}{dt^2} = 0$,

$$\frac{dP}{dr} = g(r)\rho(r) = \frac{Gm(r)\rho(r)}{r^2}$$

Question 3 (b)

$$\frac{dP_r}{dr} = \frac{dP_r}{dT}\frac{dT}{dr} = \frac{4}{3}aT^3\frac{dT}{dr}$$

$$j(r) = -\frac{4ac}{3}\frac{T^3}{\rho\kappa}\frac{dT}{dr} \quad \Rightarrow \quad \frac{dT}{dr} = -\frac{3j(r)\rho\kappa}{4acT^3}$$

Plugging in back,

$$\frac{dP_r}{dr} = \frac{4}{3}aT^3\left(-\frac{3j(r)\rho\kappa}{4acT^3}\right) = -\frac{j(r)\rho\kappa}{c}$$

But $j(r) = \frac{L}{4\pi R^2}$, therefore

$$\frac{dP_r}{dr} = \frac{L\rho\kappa}{4\pi R^2 c} = \frac{GM\rho}{R^2}$$

So the Eddington limit,

$$L_{max} = \frac{4\pi cGM}{k}$$

In a pure hydrogen system, $\kappa = 0.04$, so

$$L_{max} = \frac{4\pi cG}{0.04}M$$

$$\frac{L_{max}}{L_\odot} = 3.2 \times 10^4 \frac{M}{M_\odot}$$

If the luminosity exceeds this limit, then radiation pressure drives an outflow. It will initiate a very intense radiation-driven stellar wind from its outer layers, which means a loss of mass will occur.

Question 3 (c)

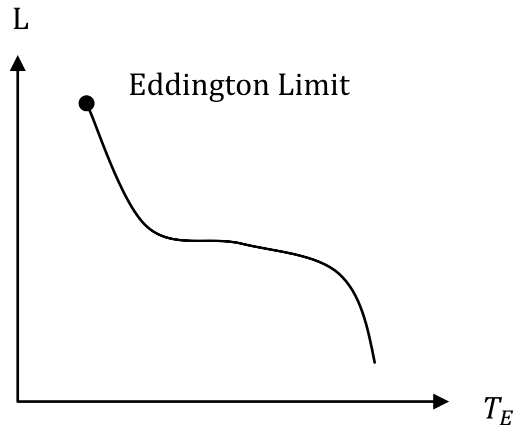
For main sequence stars,

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}$$

So

$$3.2 \times 10^4 \frac{M}{M_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5}$$

$$\therefore M = (3.2 \times 10^4)^{\frac{1}{3.5}} M_\odot$$



Solutions provided by:
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