NATIONAL UNIVERSITY OF SINGAPORE

PC3246 Nuclear Astrophysics

(Semester II: AY 2011-12)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains 3 questions and comprises 6 printed pages.
2. Answer all three questions.
3. Answers to the questions are to be written in the answer books.
4. This is a closed book examination.
5. Data and formulae are included at the end of the paper
6. One A4 page cheat sheet is allowed.

1.

a) Calculate the Q-values for the reactions:

$$^{4}He + {}^{4}He \rightarrow {}^{8}Be$$

$$^{4}He + {}^{4}He + {}^{4}He \rightarrow {}^{12}C$$

What does the difference in sign indicate?

The masses are given at the end of the paper. You may assume that:

1u=931.5 MeV/c^2.

b) At a given temperature T the fusion rate between nuclei with relative kinetic energy E is approximately proportional to:

$$\exp[-\frac{E}{kT} - \left(\frac{E_G}{E}\right)^{1/2}],$$

where k is the Boltzmann constant. The Gamow energy E_G depends upon the mass and charge of the nuclei involved. Justify the form of this expression and find the relationship between T and E_0, the value of E at which the value of the expression is a maximum.

c) Describe the CNO cycle. What is its net result in terms of burning hydrogen?

d) One reaction which takes place during the CNO cycle is:

$$^{1}H + {}^{13}C \rightarrow {}^{14}N + \gamma\,.$$

Estimate the height of the Coulomb barrier (in MeV) for this reaction and the temperature at which the nuclei will have kinetic energies of approximately this value, assuming that an element with atomic number A has a nuclear radius given by 1.2×10^{-15} A$^{1/3}$ [m].

2.

a) Consider a model of a star consisting of a spherical blackbody with a surface temperature of 18,000 K and a radius of 2×10^9 m, which is located at a distance of 50 parsecs from Earth. Calculate the value of each of the following:

i. the luminosity of the star.

ii. the radiation flux at the Earth's surface.

iii. the absolute bolometric magnitude of the star

iv. the stars apparent bolometric magnitude

b) An interstellar cloud with a mass of 100 solar masses has a temperature of 20 K and a particle density $n = 10^5$ cm^{-3}. Assume the cloud is composed entirely of H. Is the cloud stable against gravitational collapse?

c) Assuming that a star is composed of an ideal gas of uniform mean particle mass $\bar{m}$ with negligible radiation pressure, and that the gas pressure vanishes at the surface, use the equation of hydrostatic equilibrium (question 3) to show that:

$$P_c > \frac{GM^2}{8\pi R^4}$$
$$\langle T \rangle > \frac{GM\bar{m}}{6kR}$$

Here, M is the mass, R is the radius, P_c is the central pressure and $\langle T \rangle$ is the mean temperature of the star. Hint: use the virial theorem : $E_{GR} = -2E_{KE}$.

3.

a) Derive the condition of hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

for a spherical stellar system.

b) The equations for diffusive radiation transport and radiation pressure are given by:

$$j(r) = -\frac{4ac}{3}\frac{T^3}{\rho\kappa}\frac{dT}{dr}$$

$$P_r = \frac{1}{3}aT^4$$

Here, a is a constant (see data at the end of the paper), ρ is the density, κ is the opacity, c is the speed of light and T the temperature. Derive from these relationships the expression for the "Eddington limit" for stellar luminosity, valid for a system which is stabilized by radiation pressure alone, at high temperature, so that:

$$\kappa = 0.02\,(1 + X_1)\ \left[\frac{m^2}{kg}\right]$$

Briefly discuss the implications of this limit, in the case of a pure H system.

c) Assume a typical mass-luminosity relation for massive stars and estimate the upper mass limit for the main-sequence. Sketch the typical main-sequence on a labeled HR diagram and show the position of the Eddington limit.

PHYSICAL CONSTANTS AND CONVERSION FACTORS

Symbol	Description	Numerical Value
c	velocity of light in vacuum	299 792 458 m s^{-1}, exactly
μ_0	permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}
ε_0	permittivity of vacuum where $c = 1/\sqrt{\varepsilon_0\mu_0}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	gravitational constant	6.673×10^{-11} m^3 kg^{-1} s^{-2}
e	elementary charge	1.602×10^{-19} C
eV	electronvolt	1.602×10^{-19} J
α	fine structure constant, $e^2/4\pi\varepsilon_0\hbar c$	1/137.0
m_e	electron mass	9.109×10^{-31} kg
m_ec^2	electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $e\hbar/2m_e$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\alpha^2 m_ec^2/2$	13.61 eV
a_0	Bohr radius, $(1/\alpha)$ $(\hbar/m_ec)$	0.5292×10^{-10} m
Å	angstrom	10^{-10} m
m_p	proton mass	1.673×10^{-27} kg
m_pc^2	proton rest-mass energy	938.272 MeV
m_nc^2	neutron rest-mass energy	939.566 MeV
μ_N	nuclear magneton, $e\hbar/2m_p$	5.051×10^{-27} J T^{-1}
fm	femtometre or fermi	10^{-15} m
b	barn	10^{-28} m^2
u	atomic mass unit, $\frac{1}{12}m(^{12}C$ atom)	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	triple point temperature	273.16 K
k	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	molar gas constant, $N_A\kappa$	8.315 J mol^{-1} K^{-1}
σ	Stefan–Boltzmann constant, $(\pi^2/60)(\kappa^4/\hbar^3c^2)$	5.671×10^{-8} W m^{-2} K^{-4}
M_E	mass of earth	5.97×10^{24} kg
R_E	mean radius of earth	6.4×10^6 m
g	standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	standard atmosphere	101 325 Pa, exactly
$M_\odot$	solar mass	1.989×10^{30} kg
$R_\odot$	solar radius	6.960×10^8 m
$L_\odot$	solar luminosity	3.862×10^{26} W
$T_\odot$	solar effective temperature	5800 K
AU	astronomical unit, mean earth–sun distance	1.496×10^{11} m
pc	parsec	3.086×10^{16} m
y	year	3.156×10^7 s

Formulae

Stellar Magnitudes and Distances

$m_1 - m_2 = -2.5 \log_{10}(f_1 / f_2)$ $\qquad M = -2.5 \log_{10} (L / L_{\odot}) + 4.72$

Radiation

$\lambda \nu = c$, $\lambda_{\max} T = 0.0029\ mK$, $E = h\nu$, $L = 4\pi R^2 \sigma T^4$

$a = 7.6 \times 10^{-16}\ [\frac{\mathrm{J}}{\mathrm{K}^4 m^3}]$

Jeans density

$$\rho_J > \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\bar{m}}\right)^3$$

Chemical Potential (classical, non-relativistic)

$$\mu(A) = m_A c^2 - kT \ \ln\left[\frac{g_A n_{QA}}{n_A}\right] \qquad n_Q = \left[\frac{2\pi m k T}{h^2}\right]^{3/2}$$

Chemical Potential (classical, relativistic)

$$\mu(A) = -kT \ln \frac{g_A n_Q}{n_A} \qquad n_Q = 8\pi\left[\frac{kT}{hc}\right]^3$$

Born approximation scattering amplitude for Coulomb potential

$$f(\theta) = \int_0^\infty \rho(r)\ \frac{\sin(qr/\hbar)}{(qr/\hbar)} 4\pi r^2 dr \ \times \frac{2mkZe^2}{\hbar q} \int_0^\infty \sin(qs/\hbar) ds$$

Keplers third law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3$$

$m_{^4He} = 4.002603\ u$

$m_{^8Be} = 8.005305\ u$

$m_{^{12}C} = 12.00000\ u$

END OF PAPER

[T.O.]