Question 1 (a)

$T_E = 1.04 \times 10^4 \mathrm{K}$
$R = 2.6 R_\odot$
$M = 3.6 M_\odot$
$X_1 = 0.7$
$X_4 = 0.3$

i)

$L = 4\pi R^2 \sigma T_E^4$

$$M_B = -2.5 \lg\left(\frac{L}{L_\odot}\right) + 4.72$$

ii)

$$m = -2.5 \lg\left(\frac{20}{10}\right)^2 + M_B$$

iii)

$$26.7\mathrm{MeV} \times \frac{3.6\mathrm{M}_\odot}{4m_p} \times 0.15 \times 0.70$$

iv)

$$t_N = \frac{(\boldsymbol{iii})}{L}$$

Question 1 (b)

Cloud radius, $R_c = 2\mathrm{AU}$

$$\rho_J = \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\overline{m}}\right)^3$$

$M = \rho V \propto R^3$

$$\frac{M_1}{M_2} = \left(\frac{R_1}{R_2}\right)^3$$

$$\therefore M = 3.6 M_\odot \left(\frac{2\mathrm{AU}}{2.6 R_\odot}\right)^3$$

$M < \frac{3kT}{2G\overline{m}} R$ to be stable, so

$$T > \frac{2GM\overline{m}}{3kR}, \qquad \overline{m} = \frac{2m_p}{1 + 3X_1 + \frac{1}{2}X_4}$$

Question 2 (i)

Series of reaction in p-p chain,

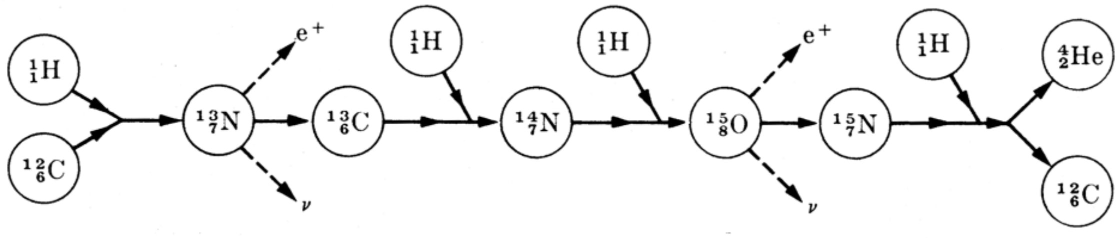
$$\left.\begin{aligned} p + p &\rightarrow d + e^+ + \nu_e \\ p + d &\rightarrow {}^3He + \gamma \end{aligned}\right\} \begin{cases} 2\,{}^3He \rightarrow {}^4He + 2p & \text{[Branch I: 85%, Q}_{\text{eff}} = 26.2\text{MeV]} \\ {}^3He + {}^4He \rightarrow {}^7Be + \gamma \begin{cases} e^- + {}^7Be \rightarrow {}^7Li + \nu_e \\ p + {}^7Li \rightarrow 2\,{}^4He \\ p + {}^7Be \rightarrow {}^8B + \gamma \\ {}^8B \rightarrow {}^8Be^* + e^+ + \nu_e \\ {}^8Be^* \rightarrow 2\,{}^4He \end{cases} & \begin{aligned} &\text{[Branch II: 15%, Q}_{\text{eff}} = 25.2\text{MeV]} \\ &\text{[Branch III: 0.02%, Q}_{\text{eff}} = 19.1\text{MeV]} \end{aligned} \end{cases}$$

Overall reaction,

$4p \rightarrow {}^4He + 2e^+ + 2\nu_e$

The slower reactions are at $p + p$, as p has to convert into n first before fusing. The energy released is approximated by the power law.

The CNO Cycle,



The overall reaction is the same as above.

Question 2 (ii)

p-p chain is dominant in main sequence stars and proto stars, when there isn't heavier elements. In heavier stars, the CNO cycle dominates, as it requires ^{12}C as a catalyst. CNO cycle is also needed to explain the luminosities of massive main sequence stars, because their luminosities are too high to be explained by the T^4 dependence of p-p chain burning.

Question 2 (iii)

$$V_{max} = \frac{zZe^2}{4\pi\epsilon_0 r_1}, \qquad z = 1, Z = 6$$

r_1 is the sum of the radii of the two molecules. So

$$r_1 = 1.2 \times 10^{-15}\left(12^{\frac{1}{3}} + 1\right)\text{m}$$

Question 3 (a)

$$\frac{N_{235}}{N_{238}} = 0.00723, \qquad \left.\frac{N_{235}}{N_{238}}\right|_{\mathrm{r-p}} = 1.6$$

$$t_{\frac{1}{2},{}^{235}U} = 7.13 \times 10^{8} \text{ years}$$

$$t_{\frac{1}{2},{}^{238}U} = 4.51 \times 10^{9} \text{ years}$$

$$N_{235} = N_{235,0}e^{-\lambda_{235}t}$$

$$N_{238} = N_{238,0}e^{-\lambda_{238}t}$$

Divide the two equations,

$$0.00723 = 1.6e^{-(\lambda_{235}-\lambda_{238})t}$$

$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$, so

$$t = -\frac{\ln\frac{0.00723}{1.6}}{\ln 2}\left(t_{\frac{1}{2},{}^{235}U} - t_{\frac{1}{2},{}^{238}U}\right)$$

Question 3 (b)

Everything in the red circle are the in the stable belt. Any stable element on the top left of the belt are created by the p-process (proton capture) only. Everything on the bottom right of the belt are created by the r-process only (rapid process). Everything on the stable belt can be created by the s-process (slow process), but some of them, if you draw a diagonal line towards the bottom right and does not hit any stable element, can also be created by the r-process. Basically r-process creates stable atoms by many beta decays (moving diagonal up the left). So the answer for this question is:

s-process : Xe-128, Xe-129, Xe-130, Xe-131, Xe-132
r-prcoess : Xe-129, Xe-131, Xe-132, Xe-134, Xe-136
s- & r-process : Xe-129, Xe-131, Xe-132
p-process : Xe-124, Xe-126 (they are shielded by the diagonal-left-up line of the r-process)

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