NATIONAL UNIVERSITY OF SINGAPORE

PC3246 Nuclear Astrophysics

(Semester II: AY 2012-13)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains 3 questions and comprises 6 printed pages.
2. Answer all three questions.
3. Answers to the questions are to be written in the answer books.
4. This is a closed book examination.
5. Data and formulae are included at the end of the paper
6. One A4 page cheat sheet is allowed.

1.

a) A star has a surface temperature of $T = 1.04 \times 10^4 K$, a radius $R = 2.6R_\odot$ and a total mass of $M = 3.6M_\odot$. Assume that 15% of its mass is in the core and available for fusion, and that the hydrogen and He mass fractions are $X_1 = 0.70$ and $X_4 = 0.30$.

i. Calculate the luminosity and the absolute magnitude of this star

ii. If the star is at distance of 20 parsec, what is its apparent magnitude?

iii. Calculate the amount of energy that is produced by hydrogen fusion during the star's main sequence lifetime. The effective Q value of hydrogen fusion is 26.7 MeV.

iv. Calculate the main sequence lifetime for this star.

b) Now assume that mass of the star is evenly distributed in a spherically symmetric cloud with a radius of 2 au. What would the temperature of this cloud have to be in order to be stable against spontaneous gravitational collapse?

2.

Discuss the factors that govern hydrogen fusion in stellar interiors:

i. Describe in reasonable detail the series of reactions that constitute the "proton-proton chain" and the "carbon-nitrogen cycle".

ii. What factors determine which of these two groups of reactions makes the predominant contribution to energy generation in a star?

iii. One reaction which takes place during the CNO cycle is:

$$^{1}H + {}^{12}C \rightarrow {}^{13}N + \gamma .$$

Estimate the height of the Coulomb barrier (in MeV) for this reaction and the temperature at which the nuclei will have kinetic energies of approximately this value, assuming that an element with atomic number A has a nuclear radius given by 1.2×10^{-15} $A^{1/3}$ [m].

3.

a) The present day value of the uranium isotopic ratio $^{235}U/^{238}U$ is 0.00723. The value of this ratio produced in the r-process is estimated to be about 1.6. Assuming a single supernova was the source of the heavy elements of the Solar System, how long ago would this event have been? The half-life of the isotope ^{235}U is 7.13×10^8 years and that of ^{238}U is 4.51×10^9 years.

b) Below is a part of the isotope chart. Black boxes are stable isotopes, and for unstable nuclei the half-lives are given. A "+" indicates β^+ decay (positron emission), otherwise the non-stable nuclei undergo β^- (electron emission) decay. Xenon has nine stable isotopes between A=124-136. Which of these isotopes are produced by (i) the s-process and (ii) the r-process? Note that some of the isotopes can be produced via both s- and r-process. Some isotopes cannot be produced by either s- or r-process. Which are those? What are possible processes for their synthesis?

Xe	+123 2 h	124	+125 17 h	126	+127 36 d	128	129	130	131	132	133 5 d	134	135 9 h	136	137 4min
I	+122 4 min	+123 13 h	+124 4 d	+125 59 d	+126 13 d	127	128 25 min	129 10^7a	130 12 h	131 8 d	132 2 h	133 21 h	134 52 min	135 7 h	136 84 s
Te	+121 17d	122	123	124	125	126	127 9 h	128	129 70 min	130	131 25 min	132 76h	133 13 min	134 42 min	135 19 s
Sb	+120 16 min	121	122 3 d	123	124 60 d	125 3 a	126 12 d	127 4 d	128 9 h	129 4 h	130 6 min	131 23 min	132 3 min	133 3 min	134 0.75s

PHYSICAL CONSTANTS AND CONVERSION FACTORS

Symbol	Description	Numerical Value
c	velocity of light in vacuum	$299\,792\,458$ m s^{-1}, exactly
μ_0	permeability of vacuum	$4\pi \times 10^{-7}$ N A^{-2}
ε_0	permittivity of vacuum where $c = 1/\sqrt{\varepsilon_0\mu_0}$	8.854×10^{-12} C^2 N^{-1} m^{-2}
h	Planck constant	6.626×10^{-34} J s
$\hbar$	$h/2\pi$	1.055×10^{-34} J s
G	gravitational constant	6.673×10^{-11} m^3 kg^{-1} s^{-2}
e	elementary charge	1.602×10^{-19} C
eV	electronvolt	1.602×10^{-19} J
α	fine structure constant, $e^2/4\pi\varepsilon_0\hbar c$	1/137.0
m_e	electron mass	9.109×10^{-31} kg
m_ec^2	electron rest-mass energy	0.511 MeV
μ_B	Bohr magneton, $e\hbar/2m_e$	9.274×10^{-24} J T^{-1}
R_∞	Rydberg energy $\alpha^2 m_ec^2/2$	13.61 eV
a_0	Bohr radius, $(1/\alpha)\,(\hbar/m_ec)$	0.5292×10^{-10} m
Å	angstrom	10^{-10} m
m_p	proton mass	1.673×10^{-27} kg
m_pc^2	proton rest-mass energy	938.272 MeV
m_nc^2	neutron rest-mass energy	939.566 MeV
μ_N	nuclear magneton, $e\hbar/2m_p$	5.051×10^{-27} J T^{-1}
fm	femtometre or fermi	10^{-15} m
b	barn	10^{-28} m^2
u	atomic mass unit, $\frac{1}{12}m(^{12}$C atom)	1.661×10^{-27} kg
N_A	Avogadro constant, atoms in gram mol	6.022×10^{23} mol^{-1}
T_t	triple point temperature	273.16 K
κ	Boltzmann constant	1.381×10^{-23} J K^{-1}
R	molar gas constant, $N_A\kappa$	8.315 J mol^{-1} K^{-1}
σ	Stefan–Boltzmann constant, $(\pi^2/60)(\kappa^4/\hbar^3c^2)$	5.671×10^{-8} W m^{-2} K^{-4}
M_E	mass of earth	5.97×10^{24} kg
R_E	mean radius of earth	6.4×10^6 m
g	standard acceleration of gravity	9.806 65 m s^{-2}, exactly
atm	standard atmosphere	101 325 Pa, exactly
$M_\odot$	solar mass	1.989×10^{30} kg
$R_\odot$	solar radius	6.960×10^8 m
$L_\odot$	solar luminosity	3.862×10^{26} W
$T_\odot$	solar effective temperature	5800 K
AU	astronomical unit, mean earth–sun distance	1.496×10^{11} m
pc	parsec	3.086×10^{16} m
y	year	3.156×10^7 s

Formulae

Stellar Magnitudes and Distances

$m_1 - m_2 = -2.5 \log_{10}(f_1 / f_2)$ $\quad\quad M = -2.5 \log_{10} (L / L_\odot) + 4.72$

Radiation

$\lambda\nu = c$, $\lambda_{\max} T = 0.0029\ mK$, $E = h\nu$, $L = 4\pi R^2 \sigma T^4$

$a = 7.6 \times 10^{-16}\ [\frac{\text{J}}{\text{K}^4 m^3}]$

Jeans density

$$\rho_J > \frac{3}{4\pi M^2}\left(\frac{3kT}{2G\bar{m}}\right)^3$$

Chemical Potential (classical, non-relativistic)

$$\mu(A) = m_A c^2 - kT \ \ln\left[\frac{g_A n_{QA}}{n_A}\right] \quad\quad n_Q = \left[\frac{2\pi mkT}{h^2}\right]^{3/2}$$

Chemical Potential (classical, relativistic)

$$\mu(A) = -kT \ln \frac{g_A n_Q}{n_A} \quad\quad n_Q = 8\pi\left[\frac{kT}{hc}\right]^3$$

END OF PAPER

[T.O.]