NATIONAL UNIVERSITY OF SINGAPORE

PC4130 Quantum Mechanics III

(Semester I: AY 2012-13)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR** questions and comprises **THREE** printed pages.
2. Answer **ALL FOUR** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. The four questions carry equal weights.
6. One Help Sheet (A4 size, both sides) is allowed for this examination.

Question 1.

Consider a particle of mass m moving in a one-dimensional potential $V(x) = \alpha|x|$, where $\alpha > 0$.

(a) Using the WKB quantization rule, predict the quantized energy eigenvalues of the system.

(b) One may estimate the energy for the ground state and the first excited state using the variational principle. Write down your trial wavefunctions for the ground state as well as the first excited state and justify your answer.

Question 2.

Consider a charged particle in a one-dimensional infinitely deep square-well potential of width L.

(a) Find how the Einstein A and B coefficients associated with the $(n+1) \to n$ transition scale with L, where n is the quantum number for the nth energy eigenstate of the system.

(b) Under the electric-dipole approximation, show that the system cannot make $(n+2k) \to n$ transitions by spontaneously emitting one photon, where k is an integer.

Question 3.

A two-level quantum system is interacting with a laser field with the Hamiltonian

$$H = \begin{pmatrix} E_1 & \hbar\Omega_p \cos(\omega_p t) \\ \hbar\Omega_p^* \cos(\omega_p t) & E_2 \end{pmatrix},$$

where E_1 and E_2 are the bare energy eigenvalues without the laser field (with $E_1 < E_2$), t is the time variable, Ω_p is proportional to the laser field strength, and ω_p is the laser frequency. We further assume the following on-resonance condition: $\omega_p = (E_2 - E_1)/\hbar$.

(a) Under the rotating wave approximation and in the "dressed-state" picture, obtain an effective time-independent Hamiltonian for this problem.

(b) Using the effective Hamiltonian obtained above, derive the time dependence of the population on the bare energy eigenstate $|E_2\rangle$ if initially the system is on the bare energy eigenstate $|E_1\rangle$. Discuss your result.

Question 4.

Briefly explain the following items.

(a) Berry phase

(b) First Born approximation

(c) Interaction representation

(d) Optical theorem

(e) Hyperfine splitting

END OF PAPER (JG)