NATIONAL UNIVERSITY OF SINGAPORE

PC4130 Quantum Mechanics III

(Semester I: AY 2016-17)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Please write your student number only. Do not write your name.
2. This assessment paper contains **4** questions and comprises **3** printed pages.
3. Students are required to answer ALL **4** questions.
4. Students should write the answers for each question on a new page.
5. This is a CLOSED BOOK examination.
6. One help sheet of two A4 pages is allowed for this examination.

Question 1.

An unperturbed quantum system is described by Hamiltonian H^0, with nondegenerate eigenvalues E_n^0 and eigenstates $|\psi_n^0\rangle$. Consider then a static perturbation V introduced to the system.

(a) Derive the first-order perturbation to the eigenstates.

(b) Derive the second-order perturbation to the eigenvalues.

Question 2.

A spin-$\frac{1}{2}$ system is subject to a rotating magnetic field in the $x - y$ plane and a static magnetic in the positive z direction. The corresponding time-dependent Hamiltonian is given by

$$H = \begin{pmatrix} -\hbar\omega_0/2 & \hbar(\Omega/2)e^{i\phi} \\ \hbar(\Omega/2)e^{-i\phi} & \hbar\omega_0/2 \end{pmatrix},$$

where $\phi = \omega t$, t is the time variable, ω the angular frequency of the rotating field, and ω_0 is the transition frequency between spin-up and spin-down states (here up and down refer to orientation along the z direction), determined by the strength of the static field.

(a) Using the first-order time-dependent perturbation theory and assuming that the rotating field is weak (Ω being small), predict the time-dependent population in the spin-down state if initially the system is prepared in the spin-up state.

(b) If ω is sufficiently small and if the initial state is prepared as the ground state of H at $t = 0$, find the time evolving state at arbitrary time $t > 0$ under the adiabatic approximation, taking into account the geometric phase and the dynamical phase.

(c) Find the Berry phase under the conditions in (b), obtained after the rotating field has rotated one cycle.

Question 3.

Consider a point particle of mass m moving in a one-dimensional potential $V(x) = \alpha|x|$, where $\alpha > 0$, and $x \in (-\infty, +\infty)$.

(a) Using the variational principle and appropriate trial wavefunctions with Gaussian decay, find the energy eigenvalues of the ground state AND the first excited state of this system.

(b) Now assume that the point particle carries one unit charge. Using results from (a), find how the spontaneous emission rate (from the first excited state to the ground state) scales with m, α, and $\hbar$. Hint: intermediate results from (a) may help you to determine the characteristic length scale of the system.

Question 4.

Briefly explain the following items.

(a) Electric-dipole selection rule

(b) Optical theorem in quantum scattering theory

(c) WKB quantization rule

(d) Interaction representation

(e) Lippmann-Schwinger equation

END OF PAPER (JG)