NATIONAL UNIVERSITY OF SINGAPORE

PC4130 Quantum Mechanics III

(Semester I: AY 2008-09)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR** questions and comprises **THREE** printed pages including this page.
2. Answer all **FOUR** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. One help sheet (A4 size, both sides) is allowed for this examination.
6. All four questions carry equal marks.

Question I.

Consider the time evolution of a quantum system with a time-varying Hamiltonian $\hat{H}(\lambda(t))$. The instantaneous eigenstates and eigenvalues of $\hat{H}(\lambda(t))$ are given by $|\psi_n(\lambda(t))\rangle$ and $E_n(\lambda(t))$. Suppose now $\lambda(t)$ is changing very slowly such that a time evolving state starting from $|\psi_n(\lambda(t_0))\rangle$ at $t = t_0$ remains to be an eigenstate of $\hat{H}(\lambda(t))$.

1. What is the dynamical phase of the time-evolving state?
2. Derive the expression for the geometric phase of the time-evolving state.

Question II.

Using the variational principle, estimate the energy of the first excited state associated with the harmonic potential $V(x) = cx^2$, where $c > 0$. You may need the following integral:

$$\int_0^{+\infty} \exp(-ax^2)x^{2n}dx = \sqrt{\frac{\pi}{4a}}\left(\frac{1}{4a}\right)^n \frac{(2n)!}{n!}.$$

Question III.

Consider the quantum scattering by a shell potential modeled by a spherical delta function, namely, $V(r) = V_0\delta(r-a)$, where r represents the distance from the origin.

1. Using the first Born approximation, calculate the differential cross section.

2. In the regime of high energy scattering, also estimate how the total cross section scales with the scattering energy.

Question IV.

The eigenvalues and eigenstates of a quantum system are given by E_n and $|\psi_n\rangle$, with $n = 1, 2, 3, \cdots$. For time $t < 0$, the system is in its ground state $|\psi_1\rangle$. For time $t \geq 0$, this system is subject to a perturbation $\hat{V}(t) = \hat{A}\exp(-t/\tau)$, where $\hat{A}$ is a time-independent operator.

1. Using the first-order time-dependent perturbation theory, obtain the final probability of finding the system being in an arbitrary excited state $|\psi_n\rangle$.

2. Discuss also under what conditions your first-order perturbation result will hold.

END OF PAPER, JG