NATIONAL UNIVERSITY OF SINGAPORE

PC4240 Solid State Physics (II)

(Semester II: AY 2012-13)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **3** questions and comprises **4** printed pages.
2. Answer all **3** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a CLOSED BOOK examination.
5. One Help Sheet (A4 size, one side) is allowed for this examination
6. A book of constant is provided.

1 (a) Consider an electron in a weak periodic potential, U(x)=U(x+a), in one dimension. The periodic potential is written as

$$\mathrm{U(x)} = \sum_G e^{iGx} U_G$$

where the sum is over the reciprocal lattice $G = \dfrac{2\pi n}{a}$ and $U_G^* = U_{-G}$ .Write the wave function as $\psi = \sum_k C(k) e^{ikx}$

The eigenvalue (ε) equation is given as

$$(\lambda_k - \varepsilon) C(k) + \sum_G U_G C(k - G) = 0$$

where $\lambda_k = \dfrac{\hbar k^2}{2m}$.

(i) Show that the electron wave function for k near to a Brillouin zone is [5 marks]

$$\psi = A e^{ikx} + B e^{i(k-G)x}$$

(ii)For an electron of mass m with k exactly at a zone boundary, show that the energy gap is $2U_G$. [7 marks]

1(b) Write down the Hamiltonian for a free electron without spin moving in a magnetic field of vector potential A=(0,Bx,0) in terms of B, the components of wave vector k, and position coordinates. [5 marks]

1 (c) In the de Haas-van Alphen effect, the total energy of the N electron system with degeneracy ρ is given as

$$U = \frac{e\hbar N^2}{2m^* c\rho}\left[1 - (1 - \frac{B}{B_s})(1 - \frac{B}{B_{s+1}})\right]$$

B_s is the critical magnetic fields of Landau level ,s ,and given as $\dfrac{1}{B_s} = \dfrac{\rho}{N} s$. Derive an expression to show that the magnetic moment μ is an oscillatory function of 1/B.

[7 marks] question to be continued

1(d) (i) Write down a macroscopic wavefunction in superconductivity. [3 marks]

(ii) Starting from the macroscopic wavefunction or otherwise, show that the magnetic flux through a superconducting ring is quantized. [7 marks]

2(a) Use Hund's rule to calculate the magnetic moment of atomic Cobalt ($3d^7$ $4s^2$) and Carbon ($2s^22p^2$). [6 marks]

2(b) (i) Consider various contributions to the domain energy, explain why a ferromagnet prefer to form domains. Sketch two possible domain structures. [4 marks]

(ii) Consider a single-domain, uniaxial saturated magnetized spherical particle with saturated magnetization Ms and diameter d. Derive an expression for the demagnetization energy(self-magnetization energy) of the sphere in terms of d and Ms. [4 marks]

(iii) Assume now that the above sphere has a single wall in an equatorial plane and forms two domains. Derive an expression of the total energy of the sphere in terms of d, Ms and domain wall energy per unit area, σ_w. [4 marks]

(iv) Give an expression of critical d in terms of σ_w and Ms below which the particle is stable as the single domain. [4 marks]

2(c) Show that the low-temperature heat capacity due to excitations which obey a dispersion relation $\omega=Ak^2$ varies as $T^{3/2}$ for a three –dimensional ferromagnet. A is a constant.

(Hint) $\int_0^\infty \frac{x^{3/2}dx}{e^x-1} = c$ = constant [11 marks]

3 (a) (i) Using the Thomas-Fermi theory, derive an expression to relate the induced electron concentration ($n(x)-n_0$) to the perturbation electrostatic potential $\phi(x)$ and Fermi energy ε_F. Assume a free electron gas with $\varepsilon_F = \frac{\hbar^2}{2m}(3\pi^2 n_0)^{2/3}$. [8 marks]

(ii) Use the screened Coulomb potential to explain the occurrence of the Mott metal-insulator transition. Distinguish between a Mott insulator and normal insulator. [5 marks]

3(b) (i) Write down the Landau free energy density expression F(P,T) close to Tc for a ferroelectric crystal for zero applied electric field for the case of second order phase transition . P is the polarization. The sign of each term has to be stated clearly. [6 marks]

(ii) Show that for a second order phase transition, the susceptibility χ of the ferroelectric below Tc is $\chi = \frac{1}{2\gamma(T_c - T)}$

γ is taken as a positive constant. [8 marks]

3(c) In an optics experiment, the reflectance $R(\omega)=r^*(\omega)r(\omega)$ is normally measured. Reflectivity $r(\omega) = \rho(\omega)\,e^{i\theta(\omega)}$ is viewed as a response function between the incident and reflected waves.The phase $\theta(\omega)$ of the reflected wave is normally obtained with the aid of the Kramers-Kronig relation. Explain with equations how $\theta(\omega)$ can be calculated from the measured reflectance $R(\omega)$ at all frequencies. Hint: Kramers-Kronig relations:

$$\alpha'(\omega) = \frac{2}{\pi} P\int_0^\infty \frac{s\alpha''(s)}{s^2-\omega^2}ds \quad \text{and} \quad \alpha''(\omega) = -\frac{2\omega}{\pi} P\int_0^\infty \frac{\alpha'(s)}{s^2-\omega^2}ds$$

[6 marks]

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