NATIONAL UNIVERSITY OF SINGAPORE

PC4240 SOLID STATE PHYSICS (II)

(Semester II: AY 2015-16)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Please write your student number only. Do not write your name.
2. This assessment paper contains **FOUR** questions and comprises **FOUR** printed pages.
3. Students are required to answer **any THREE** questions.
4. Students should write the answers for each question on a new page.
5. This is a CLOSED BOOK examination.
6. One Help Sheet (A4 size, both sides) is allowed for this examination.

1. The entropy per unit volume S_N in the normal state is related to the entropy per unit volume S_S in the superconducting state by

$$S_N = S_S - \frac{1}{8\pi}\frac{d}{dT}H_C^2 \quad ,$$

where H_C is the critical field.

(i) Show that at the transition temperature T_C the discontinuity in heat capacity per unit volume is given by

$$C_N - C_S = -\frac{T_C}{4\pi}\left(\frac{dH_C}{dT}\right)^2 .$$

(ii) The observed threshold field curve of H_C versus T_C is represented fairly well by the parabola

$$H_C(T) = H_0\left[1 - \left(\frac{T}{T_C}\right)^2\right] \quad ,$$

where H_0 is the value of the critical field at absolute zero. Show that

$$C_N - C_S = \frac{H_0^2}{2\pi T_C}\left[\frac{T}{T_C} - 3\left(\frac{T}{T_C}\right)^3\right] \quad .$$

(iii) It is known experimentally that C_S of a metal in the superconducting state does not contain a term linear in T, so that the linear term in the above equation is to be identified with the electronic contribution γT to the heat capacity of the metal in the normal state. Calculate the value of γ for Al with H_0=99G and T_C=1.18K.

2. The contribution of the partially filled electronic d-shell of N ions in a unit volume of solid to the magnetization is given by

$$M = Ng\mu_B J B_J(x), \qquad x = gJ\mu_B B/k_B T,$$

where $B_J(x)$ is the Brillouin function, g the Landé-g-factor and μ_B the Bohr magneton.

(i) Show that the magnetic susceptibility is given by

$$\chi = \frac{Np^2\mu_B^2}{3k_B T} \quad ,$$

where p is the effective number of Bohr magneton.

(Question continued on next page)

(ii) Consider Co^{2+} ion whose basic d-shell is given by $3d^7$. Assuming that the orbital angular momentum is not quenched, determine the ground-state term and the effective Bohr magneton number p. Repeat the calculations assuming the orbital angular moment is quenched. The experimental value for p is 4.8.

Comment on your calculated results.

Hint: $B_J(x) = \frac{J+1}{J}\frac{x}{3}$, $x \ll 1$.

3. The complex refractive index N and the ac conductivity $\sigma(\omega)$ for a metal with cubic symmetry are given by

$$N = n + iK = \left[1 + \frac{4\pi i}{\omega}\sigma(\omega)\right]^{\frac{1}{2}}, \qquad \sigma(\omega) = \sigma(0)\left[\frac{1+i\omega\tau}{1+\omega^2\tau^2}\right],$$

where $\sigma(0)=ne^2\tau/m$, ω is the angular frequency and τ the relaxation time.

(i) Show that the real and imaginary parts of N^2 are given by

$$n^2 - K^2 = 1 - \frac{4\pi\sigma(0)\tau}{1+\omega^2\tau^2} = 1 - \frac{\omega_p^2\tau^2}{1+\omega^2\tau^2},$$

$$2nK = \frac{4\pi\sigma(0)}{\omega(1+\omega^2\tau^2)} = \frac{\omega_p^2\tau}{\omega(1+\omega^2\tau^2)},$$

where $\omega_p=[4\pi ne^2/m]^{½}$ is the plasma frequency.

(ii) Using the relation

$$R = \frac{(n-1)^2 + K^2}{(n+1)^2 + K^2} = 1 - \frac{4n}{(n+1)^2 + K^2}$$

for the reflectance at normal incidence, verify the following expressions for R in the three given regions:

$$R \approx 1 - \sqrt{\frac{2\omega}{\pi\sigma(0)}} \qquad \omega \ll \frac{1}{\tau}$$

$$\approx 1 - \frac{2}{\omega_p\tau} \qquad \frac{1}{\tau} \ll \omega \ll \omega_p$$

$$\approx 0 \qquad \omega \gg \omega_p\ .$$

(iii) Discuss the optical properties of the metal in the above three regions.

4. The dielectric function of an ionic crystal with N ion pairs per unit volume is given by

$$\varepsilon(\omega) = \varepsilon(\infty) + \frac{4\pi N q^2}{M(\omega_T^2 - \omega^2)} \quad ,$$

where q is the effective charge and M the reduced mass.

(i) Show that the transverse optical phonon frequency ω_T is related to the longitudinal optical frequency ω_L by

$$\omega_L^2 = \omega_T^2 + \frac{4\pi N q^2}{M\varepsilon(\infty)} \quad .$$

(ii) Show that $$\varepsilon(\omega) = \frac{\omega_T^2 \varepsilon(0) - \omega^2 \varepsilon(\infty)}{\omega_T^2 - \omega^2} \quad .$$

(iii) Sketch $\varepsilon(\omega)$ versus ω and discuss the optical properties of the ionic crystal.

(iv) Verify the Lyddane-Sachs-Teller relation

$$\frac{\omega_L^2}{\omega_T^2} = \frac{\varepsilon(0)}{\varepsilon(\infty)} \quad .$$

Experimental values of static dielectric constant, optical dielectric constant and transverse optical frequency for *NaCl* crystal are 5.9, 2.25 and $3.1\mathrm{x}10^{13}$ *Hz* respectively. Determine the longitudinal optical phonon frequency.

SCNG

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