NATIONAL UNIVERSITY OF SINGAPORE

PC4242: ELECTRODYNAMICS

(Semester II: AY 2017-18)

Time allowed: 2 hours

INSTRUCTIONS TO STUDENTS

1. Please write your student number only. **Do not write your name.**
2. This assessment paper contains **TWO** questions and comprises **THREE** printed pages.
3. Students are required to answer **ALL** questions.
4. Students should write the answers for each question on a new page.
5. This is a **CLOSED BOOK** examination.
6. Some useful formulas are provided on Page 2.
7. The use of **electronic equipment** of any kind **is not permitted**.

Formulas

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} = \frac{4\pi}{c}\mathbf{J}$$

$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B} = 0$$

$$\frac{\partial}{\partial t}\rho + \nabla \cdot \mathbf{J} = 0$$

$$\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{A} - \nabla\phi$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\frac{1}{c}\frac{\partial}{\partial t}\phi + \nabla \cdot \mathbf{A} = 0$$

$$\mathbf{f} = \rho\mathbf{E} + \frac{1}{c}\mathbf{J} \times \mathbf{B}$$

$$(\eta_{\mu\nu}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = (\eta^{\mu\nu})$$

$$\begin{pmatrix} ct' \\ \mathbf{x}' \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\frac{\mathbf{v}}{c}\cdot \\ \gamma\frac{\mathbf{v}}{c} & \mathbf{1} + (\gamma - 1)\frac{\mathbf{v}\,\mathbf{v}}{v^2}\cdot \end{pmatrix} \begin{pmatrix} ct \\ \mathbf{x} \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{c}A_\mu J^\mu - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}$$

$$T^{\mu\nu} = \frac{1}{4\pi}\eta_{\alpha\beta}F^{\mu\alpha}F^{\nu\beta} - \frac{1}{16\pi}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$$

$$t_r = t_e + \frac{\mathbf{n}\cdot\mathbf{x}'}{c} = t - \frac{r}{c} + \frac{\mathbf{n}\cdot\mathbf{x}'}{c}$$

$$\frac{\mathrm{d}P(t)}{\mathrm{d}\Omega} = \frac{1}{4\pi c^3}\left|\mathbf{n} \times \int \mathrm{d}^3x'\frac{\partial}{\partial t}\mathbf{J}(\mathbf{x}', t_r)\right|^2$$

$$P(t) = \frac{2}{3}\frac{e^2}{c^3}\left|\dot{\mathbf{v}}(t_e)\right|^2$$

$$\mathbf{d}(t) = \int \mathrm{d}^3x\,\mathbf{x}\,\rho(\mathbf{x}, t)$$

$$\boldsymbol{\mu}(t) = \frac{1}{2c}\int \mathrm{d}^3x\,\mathbf{x} \times \mathbf{J}(\mathbf{x}, t)$$

$$\mathbf{Q}(t) = \int \mathrm{d}^3x\left(3\mathbf{x}\mathbf{x} - \mathbf{1}\mathbf{x}^2\right)\rho(\mathbf{x}, t)$$

$$f(t) = \int \frac{\mathrm{d}\omega}{2\pi}\mathrm{e}^{-\mathrm{i}\omega t}f(\omega)$$

$$f(\omega) = \int \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t}f(t)$$

$$f(\mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}f(\mathbf{k})$$

$$f(\mathbf{k}) = \int \mathrm{d}^3x\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}f(\mathbf{x})$$

$$\begin{pmatrix} \phi \\ \mathbf{A} \end{pmatrix}(\mathbf{x}, \omega) \approx \frac{\mathrm{e}^{ikr}}{r}\begin{pmatrix} \rho \\ \frac{1}{c}\mathbf{J} \end{pmatrix}(\mathbf{k}, \omega)$$

$$\frac{\mathrm{d}\mathcal{E}(\omega)}{\mathrm{d}\Omega} = \frac{\omega^2}{4\pi^2c^3}\left|\mathbf{n} \times \mathbf{J}(\mathbf{k}, \omega)\right|^2$$

$$\begin{pmatrix} \phi \\ \mathbf{A} \end{pmatrix}(\mathbf{x}, t) = \int \mathrm{d}^3x'\frac{1}{|\mathbf{x} - \mathbf{x}'|}\begin{pmatrix} \rho \\ \frac{1}{c}\mathbf{J} \end{pmatrix}\left(\mathbf{x}', t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}\right)$$

$$\frac{\mathrm{d}P(t)}{\mathrm{d}\Omega} = \frac{1}{4\pi c^3}\left|\mathbf{n} \times \left[\ddot{\mathbf{d}}(t_e) - \mathbf{n} \times \ddot{\boldsymbol{\mu}}(t_e) + \frac{1}{6c}\dddot{\mathbf{Q}}(t_e)\cdot\mathbf{n}\right]\right|^2$$

$$\frac{\mathrm{d}P(\omega, T)}{\mathrm{d}\Omega} = \frac{1}{4\pi^2c}\int_{-\infty}^{\infty}\mathrm{d}\tau\,\mathrm{e}^{\mathrm{i}\omega\tau}\left[\mathbf{k} \times \mathbf{J}\left(\mathbf{k}, T + \tfrac{1}{2}\tau\right)\right]\cdot\left[\mathbf{k} \times \mathbf{J}^*\left(\mathbf{k}, T - \tfrac{1}{2}\tau\right)\right]$$

$$E_x(\mathbf{x}) = -\frac{E_0}{2\pi}\frac{\mathrm{e}^{\mathrm{i}kr}}{r}\mathrm{i}k\cos\theta\int_{\text{apertures}}\mathrm{d}^2x'_\perp\,\mathrm{e}^{-\mathrm{i}k\mathbf{n}\cdot\mathbf{x}'_\perp}$$

$$\mathbf{E}(\mathbf{x}) = \mathbf{E}_{\text{inc}}(\mathbf{x}) + \mathrm{i}k\left(\mathbf{1} + \frac{\nabla\nabla}{k^2}\right)\cdot\int \mathrm{d}^2x'_\perp\frac{\mathrm{e}^{\mathrm{i}k|\mathbf{x} - \mathbf{x}'_\perp|}}{|\mathbf{x} - \mathbf{x}'_\perp|}\frac{1}{c}\mathbf{K}(\mathbf{x}'_\perp)$$

1: A point charge e undergoes a simple harmonic motion,

$$\mathbf{R}(t) = L\sin(\omega t)\mathbf{e}_z.$$

(a) Write down the charge density $\rho(\mathbf{x}, t)$ and the current density $\mathbf{J}(\mathbf{x}, t)$.

[5 marks]

(b) Calculate the electric dipole moment of the system.

[5 marks]

(c) Calculate the magnetic dipole moment of the system.

[5 marks]

(d) Calculate the electric quadrupole moment of the system.

[5 marks]

(e) Calculate the radiating power distribution $\frac{\mathrm{d}P(t)}{\mathrm{d}\Omega}$.

[10 marks]

(f) Calculate the radiating power distribution averaging over one period $\langle\frac{\mathrm{d}P}{\mathrm{d}\Omega}\rangle_t$.

[5 marks]

(g) Calculate the total radiating power averaging over one period.

[5 marks]

2: On 31 January 2018, people in Singapore observed a so-called "Super Blue Blood Moon." Here, a "super moon" means that the full moon is near to its closest point to the earth. A "blue moon" refers to the second full moon in one calendar month. What is the physical mechanism leading to the term "blood moon?" Use a simple model to derive the formulas needed.

[20 marks]

[WQh]

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