NATIONAL UNIVERSITY OF SINGAPORE

PC4245 Particle Physics

(Semester II: AY 2016-17)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Write your Matric Number on the front cover page of each answer book, do not write your name.
2. This examination paper contains **4** questions and comprises **4** printed pages. Answer any **3** questions.
3. All questions carry equal marks.
4. Students should write the answers for each question on a new page.
5. This is a CLOSED BOOK examination.

1. Define the time reversal operator U_T.

(i) Show that the time reversal operator must be antiunitary instead of unitary.

(ii) Consider a physical system having a time reversal symmetry, that is, its Hamiltonian commutes with the time reversal operator U_T. Kramer's theorem states that for such a system, the energy value of its state satisfying $|\psi\rangle = -U_T^{\,2}\,|\psi\rangle$ is doubly degenerate. Prove this theorem.

(iii) Show that nonzero electric dipole moment of an electron can lead to violation of time reversal symmetry.

Note: The following relation can be used without proof:

Using the Wigner-Eckart theorem, we have in the standard notations

$$\langle Ejm|\underset{\sim}{d}|Ejm\rangle = C_{Ej}\langle Ejm|\underset{\sim}{J}|Ejm\rangle$$

where C_{Ej} is independent of the quantum number m. Here $\underset{\sim}{d}$ is the electric dipole moment and $\underset{\sim}{J}$ the angular momentum.

2. (a) Show that the helicity operator $s(\underset{\sim}{p}) \equiv \underset{\sim}{\Sigma}\cdot\underset{\sim}{p}\,/\,|\,\underset{\sim}{p}\,|$ commutes with the Hamiltonian of a Dirac particle, $H = c\underset{\sim}{\alpha}\cdot\underset{\sim}{p} + \beta mc$. Here $\underset{\sim}{\Sigma}$ is the spin operator of the Dirac particle.

Explain qualitatively why the helicity of a particle is, in general, not an invariant.

Note: $\underset{\sim}{\Sigma} = \begin{pmatrix} \underset{\sim}{\sigma} & 0 \\ 0 & \underset{\sim}{\sigma} \end{pmatrix}$, $\underset{\sim}{\alpha} = \begin{pmatrix} 0 & \underset{\sim}{\sigma} \\ \underset{\sim}{\sigma} & 0 \end{pmatrix}$, $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and $\underset{\sim}{\sigma}$ are the Pauli matrices, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

(b) For a massive fermion, show that handedness is not a good number. That is, show that γ^5 does not commute with H.

Note: $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(c) Describe briefly two experimental evidences that suggest each quark flavour must come in three colour varieties.

3. The amplitude M for electron-muon scattering in the usual notations is given by

$$M = -\frac{g_e^2}{(\underline{p}_1 - \underline{p}_3)^2}\,[\bar{u}^{(s_3)}(\underline{p}_3)\,\gamma^\mu u^{(s_1)}(\underline{p}_1)]\,[\bar{u}^{(s_4)}(\underline{p}_4)\,\gamma_\mu u^{(s_2)}(\underline{p}_2)]$$

where g_e the dimensionless coupling constant, $\underline{p}_1$ and $\underline{p}_2$ the respective 4-momenta of the incident electron and incident muon, and $u^{(s)}(\underline{p})$ the spinors.

Evaluate amplitude M explicitly in the center-of-momentum (CM) frame, assuming the electron e^- and muon μ^- approach one another along the x^3-axis, repel, and return along the x^3-axis. The initial and final particles all have helicity **- 1**.

Notes :

(i) $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$. $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(ii) The Dirac spinor can be written as

$$u(\underline{p}) = \sqrt{(p^0 + mc)}\begin{pmatrix} W \\ \dfrac{\underset{\sim}{\sigma}\cdot\underset{\sim}{p}}{p^0 + mc}W \end{pmatrix}.$$

For negative helicity, $\underset{\sim}{\sigma}\cdot\underset{\sim}{n}\,W = -W$, where $\underset{\sim}{n} = (n^1, n^2, n^3)$ is a unit vector, the W is either given by

$$W = \frac{1}{\sqrt{2(1+n^3)}}\begin{pmatrix} n^1 - in^2 \\ -1 - n^3 \end{pmatrix} \quad \text{or} \quad W = \frac{1}{\sqrt{2(1-n^3)}}\begin{pmatrix} n^3 - 1 \\ n^1 + in^2 \end{pmatrix}.$$

4 (a) Draw a one-loop Feynman diagram (vacuum polarization) for the electron muon scattering $e^- + \mu^- \rightarrow e^- + \mu^-$.

Derive the scattering amplitude M for the one-loop diagram of the above process, using the Feynman rules for quantum electrodynamics.

Note: For vertex, $ig\gamma^\mu$; for propagators, $\dfrac{-ig^{\mu\nu}}{\underline{q}^2}$, $\dfrac{i}{q_\mu\gamma^\mu - mc}$.

(b) Using renormalization procedure, show that the scattering amplitude for the above process up to and including the one-loop diagram for vacuum polarization is given by

$$M = -\frac{g_R^2(t)}{(\underline{p}_1 - \underline{p}_3)^2} [\bar{u}^{(s_3)}(\underline{p}_3)\, \gamma^{\mu}\, u^{(s_1)}(\underline{p}_1)]\, [\bar{u}^{(s_4)}(\underline{p}_4)\, \gamma_{\mu}\, u^{(s_2)}(\underline{p}_2)] \; .$$

Here $g_R(t)$ is the renormalized coupling constant, $t = (\underline{p}_1 - \underline{p}_3)^2$ is the momentum transfer square. The following can be assumed without proof,

$$I_{\mu\nu} = \int \frac{d^4 k}{(2\pi)^4} Tr[\, \gamma_\mu \frac{1}{p_1 - p_3 - \not{k} - mc} \gamma_\nu \frac{1}{p_1 - mc}]$$

$$= \frac{i\, g_{\mu\nu}\, t}{12\pi^2} \left(\ln \frac{M^2}{m^2} - f(\frac{-t}{m^2 c^2}) \right)$$

where M is the cut-off and $f(\frac{-t}{m^2c^2})$ a finite function in the variable t .

(c) The cross section for the quark pair production process $e^- + e^+ \rightarrow \gamma \rightarrow q + \bar{q}$ in the usual notations is given by

$$\sigma = \frac{\pi\, Q^2}{3} \left(\frac{\hbar c \alpha}{E} \right)^2 \sqrt{\frac{1-(Mc^2/E)^2}{1-(mc^2/E)^2}} \left[1 + \frac{1}{2}\left(\frac{Mc^2}{E} \right)^2 \right] \left[1 + \frac{1}{2}\left(\frac{mc^2}{E} \right)^2 \right] .$$

Show that the ratio of the rate of hadron production to that of muon pairs as denoted by

$$R \equiv \frac{\sigma(e^- e^+ \rightarrow hadrons)}{\sigma(e^- e^+ \rightarrow \mu^- \mu^+)}$$

can be approximated by $R = 3 \sum_i Q_i^2$.

Deduce that $R = 2$ at energy values where only the u , d and s quarks contribute.

(OCH)

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