NATIONAL UNIVERSITY OF SINGAPORE

PC4245 Particle Physics

(Semester II: AY 2017-18)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Write your Matric Number on the front cover page of each answer book, do not write your name.
2. This examination paper contains **4** questions and comprises **5** printed pages. Answer any **3** questions.
3. All questions carry equal marks.
4. Students should write the answers for each question on a new page.
5. This is a CLOSED BOOK examination.

1. **(a)** Write down an expression of the excess energy available for inelastic scattering of collision of two particles.

The first man-made Ω^- was created by firing a high energy proton at a stationary hydrogen atom to produce a K^+/K^- pair: $p+p \rightarrow p+p+K^+ + K^-$; the K^- in turn hits another stationary proton, $K^- + p \rightarrow \Omega^- + K^+ + K^0$. What minimum kinetic energy is required (for the incident proton) to produce an Ω^- in this way?

(b) Consider the pair annihilation process $e^- + e^+ \rightarrow \gamma + \gamma$ in the lab frame of the electron (e^- at rest). Show that the differential cross section, in the usual notations, can be written as

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi}\right)^2 \frac{S|M|^2 \left|\underset{\sim}{p}_3\right|}{m_e \left|\underset{\sim}{p}_1\right| (E_1 + m_e c^2 - \left|\underset{\sim}{p}_1\right| c\cos\theta)} .$$

What is the value of S?

Note the following formula can be used

$$d\sigma = |M|^2 \frac{\hbar^2}{4}\left[(\underline{p}_1 \cdot \underline{p}_2)^2 - (m_1 m_2 c^2)^2\right]^{-\frac{1}{2}} \frac{d^3 \underset{\sim}{p}_3}{(2\pi)^3 2p_3^0} \frac{d^3 \underset{\sim}{p}_4}{(2\pi)^3 2p_4^0} (2\pi)^4 \delta^{(4)}(\underline{p}_1 + \underline{p}_2 - \underline{p}_3 - \underline{p}_4)$$

2. **(a)** Consider the weak decay of a charged pion into a muon and neutrino

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

Sketch the decay process under a mirror reflection and hence show that the decay violates the parity conservation law.

Sketch the decay process under a combination of space inversion and charge conjugation and hence show that the decay conserves the CP.

(b) Briefly describe the CP violation experiment in K^0 decay.

Find the ratio of K_S (K short) and K_L (K long) in a beam of 10 GeV/c neutral kaons at a distance of 20 meters from where the beam is produced.

(τ for $K_L = 5 \times 10^{-8}$ sec, τ for $K_S = 0.86 \times 10^{-10}$ sec)

(c) The neutral K-meson states $|K^0\rangle$ and $|\bar{K}^0\rangle$ can be expressed in terms of states $|K_L\rangle$, $|K_S\rangle$:

$$|K^0\rangle = \tfrac{1}{\sqrt{2}}(|K_L\rangle + |K_S\rangle),$$

$$|\bar{K}^0\rangle = \tfrac{1}{\sqrt{2}}(|K_L\rangle - |K_S\rangle).$$

$|K_L\rangle$ and $|K_S\rangle$ are states with definite lifetimes $\tau_L \equiv \dfrac{1}{\gamma_L}$ and $\tau_S \equiv \dfrac{1}{\gamma_S}$, and distinct rest energies $m_Lc^2 \neq m_Sc^2$. At time $t = 0$, a meson is produced in the state $|\psi(t=0)\rangle = |K^0\rangle$. Let the probability of finding the system in state $|K^0\rangle$ at time t be $P_0(t)$ and that of finding the system in state $|\bar{K}^0\rangle$ at time t be $\bar{P}_0(t)$. Find an expression for $P_0(t) - \bar{P}_0(t)$ in terms of γ_L, γ_S, m_Lc^2 and m_Sc^2. (Neglect CP violation).

3. Consider the pair annihilation process $e^- + e^+ \rightarrow \gamma + \gamma$, and assume the electron and the positron are at rest. Draw the lowest order Feynman diagrams.

Show that the scattering amplitude M can be written as

$$M = \frac{g^2}{mc}\, \bar{v}(2) \left[\underset{\sim}{\varepsilon}^*(3) \cdot \underset{\sim}{\varepsilon}^*(4)\, \gamma^0 + i(\underset{\sim}{\varepsilon}^*(3) \times \underset{\sim}{\varepsilon}^*(4)) \cdot \underset{\sim}{\Sigma}\, \gamma^3 \right] u(1)$$

If furthermore electron and positron are in a singlet state, show that the above amplitude becomes

$$M = 2\sqrt{2}\, g^2\; i\left(\underset{\sim}{\varepsilon}^*(4) \times \underset{\sim}{\varepsilon}^*(3) \right)^3 \quad .$$

Notes:

(i) $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \Sigma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix},$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

(ii) The Dirac bispinor can be written as

$$\underset{-}{u(p)} = \sqrt{(p^0 + mc)} \begin{pmatrix} W \\ \dfrac{\vec{\sigma} \cdot \vec{p}}{p^0 + mc} W \end{pmatrix}$$

where $W = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ for the particle in spin-up state and $W = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for the particle in spin-down state.

For the spin-up electron, $\underset{-}{u(p)} = \sqrt{(2mc)} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, and for the spin-down positron,

$$\underset{-}{v(p)} = \sqrt{(2mc)} \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} .$$

(iii) For vertex, $ig\gamma^\mu$; for propagators, $\dfrac{-ig^{\mu\nu}}{\underset{-}{q}^2}$, $\dfrac{i}{q_\mu \gamma^\mu - mc}$.

4. (a) Draw the lowest-order Feynman diagrams for the electron-electron scattering

$$e^- + e^- \rightarrow e^- + e^- .$$

Using the Feynman rules for quantum electrodynamics, obtain the scattering amplitude M for the above process.

Sketch also a one-loop diagram due to vacuum polarization.

Note: For vertex, $ig\gamma^\mu$; for propagators, $\frac{-ig^{\mu\nu}}{\underline{q}^2}$, $\frac{i}{q_\mu\gamma^\mu - mc}$

(b) Consider the electron-electron scattering at very high energy so that the mass of the electron can be ignored (i.e., set m = 0).

Define the spin-averaged quantity $\left\langle |M|^2 \right\rangle$.

The scattering amplitude M can be written as $M = M_1 + M_2$.

Using the Casimir trick, show that

(i) $<|M_1|^2> = \frac{g^4}{4(\underline{p}_1 - \underline{p}_3)^4} Tr(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3) \cdot Tr(\gamma_\mu \not{p}_2 \gamma_\nu \not{p}_4)$,

(ii) $<|M_1 M_2^*|> = \frac{-g^4}{16\,(\underline{p}_1 \cdot \underline{p}_3)(\underline{p}_1 \cdot \underline{p}_4)} \; Tr(\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_4 \gamma_\mu \not{p}_2 \gamma_\nu \not{p}_3)$.

Hence or otherwise obtain an expression of $\left\langle |M|^2 \right\rangle$.

Notes:

(i) For massless particles, the conservation of momentum $(\underline{p}_1 + \underline{p}_2 = \underline{p}_3 + \underline{p}_4)$ implies that

$\underline{p}_1 \cdot \underline{p}_2 = \underline{p}_3 \cdot \underline{p}_4$, $\underline{p}_1 \cdot \underline{p}_3 = \underline{p}_2 \cdot \underline{p}_4$, and $\underline{p}_1 \cdot \underline{p}_4 = \underline{p}_2 \cdot \underline{p}_3$.

(ii) $\sum_s u^{(s)}(\underline{p})\, \bar{u}^{(s)}(\underline{p}) = \not{p} + mc$, $\not{p} \equiv \gamma^\mu p_\mu$.

(OCH)

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