This document lacks answers for certain questions. Would you like to help us complete them? If yes, Please send your suggested answers to nus.physoc@gmail.com. Thanks! ☺

Question 1 (a)

$$S = \frac{1}{2!}$$

$$d\sigma = \frac{|M|^2}{2}\left(\frac{\hbar}{8\pi}\right)^2 \left[(P_1 \cdot P_2) - (m_1 m_2 c^2)^2\right]^{-1/2} \frac{d^3P_3}{|P_3|}\frac{d^3P_4}{|P_4|}\,\delta^{(4)}(P_1 + P_2 - P_3 - P_4)$$

$$\left[(P_1 \cdot P_2) - (m_1 m_2 c^2)^2\right] = (E_1 + E_2)|\vec{P}_1|c$$

$$= \int \frac{|M|^2}{2}\left(\frac{\hbar}{8\pi}\right)^2 \frac{1}{(E_1+E_2)|\vec{P}_1|c}\frac{d^3P_3}{|P_3|}\frac{d^3P_4}{|P_4|}\,\delta\left(\frac{E_1}{c} + \frac{E_2}{c} - |P_3| - |P_4|\right)\delta^{(3)}(\underbrace{P_1 + P_2}_{0} - P_3 - P_4)$$

$$\frac{d\sigma}{d\Omega} = \int \frac{|M|^2}{2}\left(\frac{\hbar}{8\pi}\right)^2 \frac{1}{2E|P_1|c}\frac{|\vec{P}_3|^2\,dP_3}{|P_3|^2}\,\delta\left(\frac{E_1}{c} + \frac{E_2}{c} - 2|P_3|\right)$$

$$=$$

$$c|P| = \sqrt{E^2 + m^2c^4}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{\hbar}{8\pi}\right)^2 \frac{|M|^2}{2E\sqrt{E^2+m^2c^4}}$$

ii) $|P_3| = |P_4| = \frac{E}{c}$

Question 1 (b)

Electron-positron annihilation process in CM frame into three photons:

$e^+ + e^- \rightarrow \gamma + \gamma + \gamma$

1 2 3 4 5

In CM frame, $E_1 = E_2 = \sqrt{m_e^2c^4 + |\vec{p}_1|^2c^2} = E_i$ (say). When all 3 final photons have the same energy, we have:

$E_3 = |\vec{p}_3|c = e_4 = |\vec{p}_4|c = E_5 = |\vec{p}_5|c = (\text{say})E_f = |\vec{p}_f|c$

Conservation of total energy then requires: $2E_i = 3E_f$.

Writing down the four-momentum of each particle:

$P_1 = \left(\frac{E_i}{c}, \vec{p}_1\right)$; $P_2 = \left(\frac{E_i}{c}, -\vec{p}_1\right)$; $P_3 = \left(\frac{E_f}{c}, \vec{p}_3\right)$; $P_4 = \left(\frac{E_f}{c}, \vec{p}_4\right)$; $P_5 = \left(\frac{E_f}{c}, \vec{p}_5\right)$

From the conservation of 4-momentum, we have:

$P_1 + P_2 = P_3 + P_4 + P_5$

Taking the dot product of itself on both sides:

$$m_e^2c^2 + m_e^2 + 2\left(\frac{E_i^2}{c^2} + \vec{p}_1^{\,2}\right) = 2(P_3 \cdot P_4 + P_3 \cdot P_5 + P_4 \cdot P_5)$$

$$2m_e^2 + 2\left(\frac{E_i^2}{c^2} + \frac{E_i^2 - m_e^2 c^4}{c^2}\right) = 6\frac{E_f^2}{c^2} - 2\vec{p}_3 \cdot (\vec{p}_4 + \vec{p}_5) - 2\vec{p}_4 \cdot \vec{p}_5$$

$$4\frac{E_i^2}{c^2} = 8\frac{E_f^2}{c^2} - 2\frac{E_f^2}{c^2}\cos\theta_{45}\ (\because \vec{p}_4 + \vec{p}_5 = -\vec{p}_3)$$

where θ_{45} is the angle between photon 4 and photon 5.

Since $2E_i = 3E_f$, this simplifies to:

$\cos\theta_{45} = -?,\qquad \text{i.e.}\ \theta_{45} = 120°.$

Similarly, we have:

$$\vec{p}_3 \cdot \vec{p}_5 = -(\vec{p}_4 + \vec{p}_5) \cdot \vec{p}_5 = -\frac{E_f^2}{c^2}(\cos\theta_{45} + 1) = -\frac{E_f^2}{2c^2}$$

$$\cos\theta_{35} = \frac{\vec{p}_3 \cdot \vec{p}_5}{E_f^2}c^2 = -?$$

Same thing for θ_{34}.

Hence $\theta_{34} = \theta_{35} = \theta_{45} = 120°$ (Q.E.D.).

Question 1 (c)

Invariant quantities will not change regardless of any ref frame used in a scattering event. (Energy)

Conserved quantities does not change in a ref frame but may change when measured in another ref frame (momentum)

Question 2 (a)

(i) First find all isospin combinations for all $p\pi$ initial states and for all ΣK final states.

$$\pi^+ + p:\ |1\ 1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \left|\frac{3}{2}\ \frac{3}{2}\right\rangle$$

$$\pi^0 + p:\ |1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$$

$$\pi^- + p:\ |1 - 1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle$$

$$\Sigma^+ + \kappa^+\ \&\ \Sigma^+ + \bar{\kappa}^0:\ |1\ 1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \left|\frac{3}{2}\ \frac{3}{2}\right\rangle$$

$$\Sigma^0 + \kappa^+\ \&\ \Sigma^0 + \bar{\kappa}^0:\ |1\ 0\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$$

$$\Sigma^- + \kappa^+ \,\&\, \Sigma^- + \bar{\kappa}^0\!:\ |1-1\rangle\left|\frac{1}{2}\ \frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left|\frac{3}{2} - \frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2} - \frac{1}{2}\right\rangle$$

$$\Sigma^+ + \kappa^0 \,\&\, \Sigma^+ + \kappa^-\!:\ |1\ 1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left|\frac{3}{2}\ \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2}\ \frac{1}{2}\right\rangle$$

$$\Sigma^0 + \kappa^0 \,\&\, \Sigma^0 + \kappa^-\!:\ |1\ 0\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}\ -\frac{1}{2}\right\rangle + \frac{1}{\sqrt{3}}\left|\frac{1}{2} - \frac{1}{2}\right\rangle$$

$$\Sigma^- + \kappa^0 \,\&\, \Sigma^- + \kappa^-\!:\ |1\ -1\rangle\left|\frac{1}{2}\ -\frac{1}{2}\right\rangle = \left|\frac{3}{2}\ -\frac{3}{2}\right\rangle$$

By observation, there are only five such processes $p\pi \to \Sigma K$ compatible with isospin conservation and charge conservation:

$A.\ \pi^+ + p \to \Sigma^+ + \kappa^+$
$B.\ \pi^0 + p \to \Sigma^0 + \kappa^+$
$C.\ \pi^0 + p \to \Sigma^+ + \kappa^0$
$D.\ \pi^- + p \to \Sigma^- + \kappa^+$
$E.\ \pi^- + p \to \Sigma^0 + \kappa^0$

(ii) The total isospin can be $\frac{3}{2}$ or $\frac{1}{2}$. So there are just two distinct amplitudes here:
M_3, for $I = \frac{3}{2}$, and M_1, for $I = \frac{1}{2}$.

$$M_A = M_3$$

$$M_B = \frac{2}{3}M_3 + \frac{1}{3}M_1$$

$$M_C = \frac{\sqrt{2}}{3}M_3 - \frac{\sqrt{2}}{3}M_1$$

$$M_D = \frac{1}{3}M_3 + \frac{2}{3}M_1$$

$$M_E = \frac{\sqrt{2}}{3}M_3 - \frac{\sqrt{2}}{3}M_1$$

The cross sections, then, stand in the ratio

$$\sigma_A\!:\sigma_B\!:\sigma_C\!:\sigma_D\!:\sigma_E = |M_3|^2\!:\frac{1}{9}|2M_3 + M_1|^2 : \frac{2}{9}|M_3 - M_1|^2\!:\frac{1}{9}|M_3 + 2M_1|^2\!:\frac{2}{9}|M_3 - M_1|^2$$

When $I = \frac{3}{2}$ channel dominates, $M_3 \gg M_1$, and hence

$$\sigma_A\!:\sigma_B\!:\sigma_C\!:\sigma_D\!:\sigma_E = |M_3|^2\!:\frac{4}{9}|M_3|^2 : \frac{2}{9}|M_3|^2\!:\frac{1}{9}|M_3|^2\!:\frac{2}{9}|M_3|^2 = 9\!:4\!:2\!:1\!:2$$

Question 2 (b) (i)

$d = n + p.$, $I = 1, 0$

when $I = 0$ $|0\,0\rangle = \frac{1}{\sqrt{2}}(np - pn)$

When $I = 1$,
$|1,1\rangle = pp$
$|1,0\rangle = \frac{1}{\sqrt{2}}(pn - np)$
$|1,-1\rangle = nn$
} bound state of 2n or 2p does not exist

$\therefore I_d = 0$

Question 2 (b) (ii)

$$\frac{\text{Rate}(n+n \to d+\pi^0)}{\text{Rate}(p+n \to d+\pi^+)} = |1,-1\rangle$$

Question 2 (c)

$P_i = -1 \qquad J_i = 0 = J_f.$

$P_f = P_1 P_2 (-1)^L$ Since pions are spinless $J = L$. and $P_\pi = -1$

$= +1$

For strong interaction to occur, parity must be conserved.

$P_f = P_1 P_2 P_3 (-1)^L$

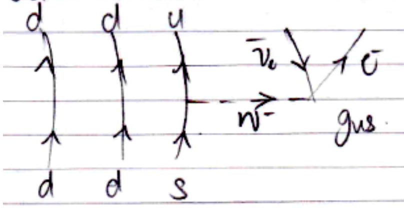
$= -1$

Therefore, η can only decay strongly into 3 pions due to parity invariance.

Question 3 (a)

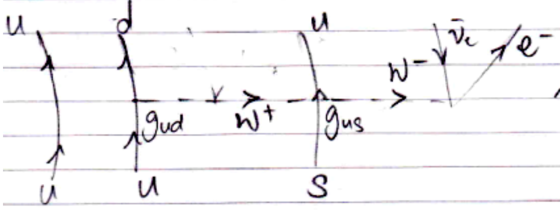
Σ^-

$dds \to udd + e^- + \bar{\nu}_e$



Σ^+

$uus \to udd + e^- + \bar{\nu}_e$



$\sim \frac{|g_{us}|^2}{|g_{ud}|^2|g_{us}|^2} = \frac{1}{|g_{ud}|^2}$

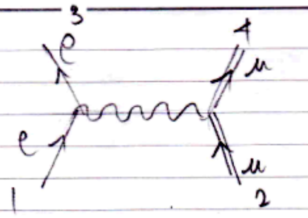
Question 3 (b)

$$\sim \frac{|g_{cs}|^2}{|g_{cd}|^2} = \frac{\cos^2\theta_c}{\sin^2\theta_c} = \frac{1}{\tan^2\theta_c} = 20$$

Decay of $c \to s$ is 20 times more likely than $c \to d$

Question 3 (c)

Question 4 (a)



$$e + \mu^- \to e + \mu^-$$
$$1 \quad\; 2 \qquad\; 3 \quad\; 4$$

$$-iM = \int \frac{d^4q}{(2\pi)^4}\, \bar{u}(3)(ig_e\gamma^\mu)u(1)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}(4)(ig_e\gamma^\nu)u(2)$$
$$\delta^{(4)}(P_1 - q - P_3)\,\delta^{(4)}(P_2 + q - P_4)$$

$$M = \underbrace{\frac{-g_e^2}{(P_1-P_3)^2}}_{A}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)]$$

$$MM^* = A^2 \frac{[\bar{u}(3)\gamma^\mu u(1)]}{[\bar{u}(1)\gamma^\nu u(3)]}[\bar{u}(4)\gamma_\mu u(2)][\bar{u}(2)\gamma_\nu u(4)]$$

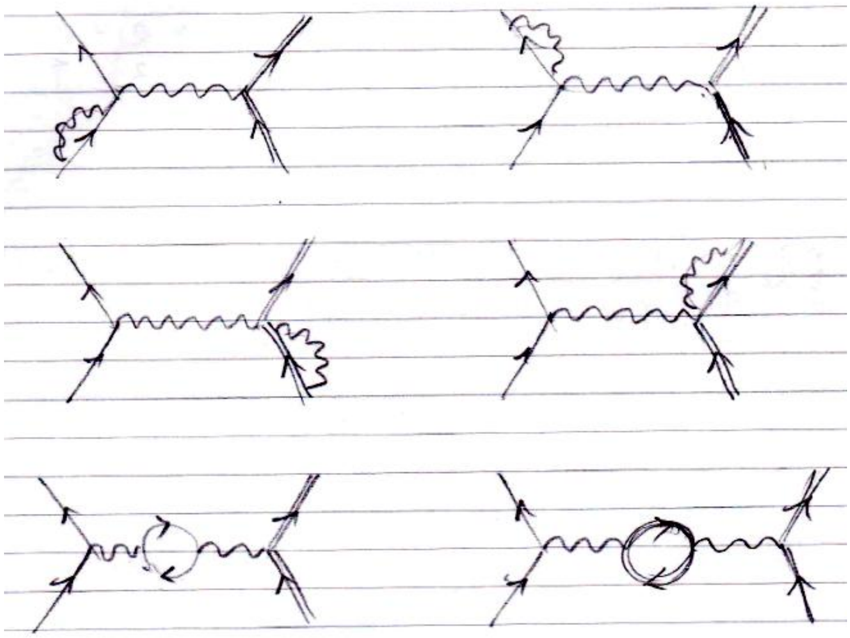
$$\langle MM^* \rangle = \frac{A^2}{4}\, \frac{\mathrm{Tr}[\gamma^\mu(\not{P}_1+m_1c)\gamma^\nu(\not{P}_3+m_3c)]}{\mathrm{Tr}[\gamma_\mu(\not{P}_2+m_2c)\gamma_\nu(\not{P}_4+m_4c)]}$$

$$\mathrm{Tr}[\gamma^\mu(\not{P}_1+m_1c)\gamma^\nu(\not{P}_3+m_3c)]$$
$$= \mathrm{Tr}[\gamma^\mu\not{P}_1\gamma^\nu\not{P}_3] + m_3c\,\underbrace{\mathrm{Tr}[\gamma^\mu\not{P}_1\gamma^\nu]}_{0} + m_1c\,\underbrace{\mathrm{Tr}[\gamma^\mu\gamma^\nu\not{P}_3]}_{0}$$
$$+ m_1m_3c^2\,\mathrm{Tr}[\gamma^\mu\gamma^\nu]$$
$$= (P_1)_\lambda(P_3)_\kappa\,\mathrm{Tr}(\gamma^\mu\gamma^\lambda\gamma^\nu\gamma^\kappa) + 4g^{\mu\nu}m_e^2c^2$$
$$= 4(P_1)_\lambda(P_3)_\kappa(g^{\mu\lambda}g^{\nu\kappa} - g^{\mu\nu}g^{\lambda\kappa} + g^{\mu\kappa}g^{\lambda\nu}) + 4g^{\mu\nu}m_e^2c^2$$
$$= 4\left((P_1)^\mu(P_3)^\nu - (P_1\cdot P_3)g^{\mu\nu} + P_1^\nu P_3^\mu + g^{\mu\nu}m_e^2c^2\right)$$

$$\mathrm{Tr}[\gamma_\mu(\not{P}_2+m_2c)\gamma_\nu(\not{P}_4+m_4c)]$$
$$= 4\left(P_{2\mu}P_{4\nu} - (P_2\cdot P_4)g_{\mu\nu} + P_{2\nu}P_{4\mu}\right) + 4g_{\mu\nu}m_\mu^2c^2)$$
$$+ (P_1\cdot P_4)(P_2\cdot P_3) - (P_1\cdot P_3)(P_2\cdot P_4)] + (P_1\cdot P_2)(P_3\cdot P_4) + (P_1\cdot P_3)(m_\mu c)^2$$
$$-2(P_2\cdot P_4)(m_ec)^2 + 4(m_em_\mu c)^2$$

$$16[(P_1\cdot P_2)(P_3\cdot P_4) - (P_2\cdot P_4)(P_1\cdot P_3) + (P_1\cdot P_4)(P_2\cdot P_3)$$
$$+ (P_1\cdot P_3)(m_\mu c)^2 \qquad + 2(m_em_\mu c)^2$$
$$-(P_1\cdot P_3)(P_2\cdot P_4) + 4(P_1\cdot P_3)(P_2\cdot P_4) - (P_1\cdot P_3)(P_2\cdot P_4) \qquad (P_2\cdot P_4)(m_ec)^2$$
$$-4(P_1\cdot P_3)(m_\mu c)^2 = 32[(P_1\cdot P_2)(P_3\cdot P_4) + (P_1\cdot P_4)(P_2\cdot P_3) - (P_1\cdot P_3)(m_\mu c)^2 -$$

Question 4 (b)



Question 4 (c)

Solutions provided by:
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