This document lacks answers for certain questions. Would you like to help us complete them? If yes, Please send your suggested answers to nus.physoc@gmail.com. Thanks! ☺

Question 1 (a) (i)

$$d\Gamma = \frac{|M|^2}{2(4\pi)^2\hbar m}\frac{d^3\vec{P}_2}{P_2^0}\frac{d^3\vec{P}_3}{P_3^0}\,\delta^{(4)}(P_1 - P_2 - P_3) \qquad S=1$$

$$= \int \frac{|M|^2}{2(4\pi)^2\hbar m_1}\frac{d^3P_2\, d^3P_3}{P_2^0\, P_3^0}\,\delta\left(m_1c - \frac{E_2}{c} - \frac{E_3}{c}\right)\delta(P_2 - P_3)$$

$$= \int \frac{|M|^2}{2(4\pi)^2\hbar m_1}\frac{|P_2|^2 dP_2\, d\Omega}{P_2^0\, P_3^0}\,\delta\left(m_1 c - \frac{E}{c}\right)$$

$$E = E_2 + E_3$$

$$= \sqrt{c^2|P_2|^2 + m_2^2c^4} + \sqrt{c^2|P_2|^2 + m_3^2c^4}$$

$$\frac{dE}{dP_2} = \frac{|P_2|c^2}{E_2} + \frac{|P_2|c^2}{E_3}$$

$$= \frac{E\,|P_2|c^2}{E_2E_3}$$

$$dE\left(\frac{E_2E_3}{E|P_2|c^2}\right) = dP_2$$

$$= \int \frac{|M|^2}{2(4\pi)^2\hbar m_1}\frac{|P_2|\, dE\, d\Omega}{E}\,\delta\left(m_1 c - \frac{E}{c}\right)$$

$$\int \frac{|M|^2}{2(4\pi)^2\hbar m_1}\frac{|P_2|c}{mc^2} \Rightarrow \Gamma = \frac{|M|^2\,|P_2|}{8\pi\hbar m_1^2 c}$$

Question 1 (a) (ii)

$$mc^2 = \underbrace{(c^2|P_2|^2 + m_2^2c^4)^{1/2}}_{E_2} + \underbrace{(c^2|P_2|^2 + m_3^2c^4)^{1/2}}_{E_3}$$

$$(mc^2 - E_3)^2 = c^2|P_2|^2 + m_2^2c^4$$

$$m_1^2c^4 + E_3^2 - 2mc^2E_3 = c^2|P_2|^2 + m_2^2c^4$$

$$m_1^2c^4 + m_3^2c^4 - m_2^2c^4 = 2m_1c^2E_3$$

$$\frac{1}{2m_1}\left[m_1^2 + m_3^2 - m_2^2\right]c^2 = E_3$$

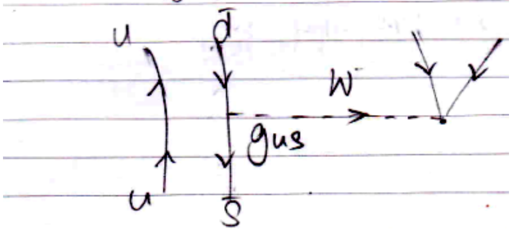
$$\frac{1}{4m_1^2}\left(m_1^2 + m_3^2 - m_2^2\right)^2c^4 = c^2|P_2|^2 + m_3^2c^4 \Rightarrow \text{solve for } |P_2|$$

Question 1 (b)

- Electric dipole moment ($\vec{d}(t)$)
- Quantities that are precisely zero when reaction is reversed

Question 2 (a) (i)

$u\bar{s} \to u\bar{d}\,\nu\bar{\nu}$



Question 2 (a) (ii)

$\pi^0 \to \gamma\gamma\gamma$

$C_i = (-1)^{L+S} = (-1)^0 = 1$

$C_f = (-1)^{n=3} = -1$

1st reaction violates charge conjugation.

$C_f = (-1)^2 = 1$

Question 2 (a) (iii)

Question 2 (a) (iv)

$J_i = 0 = J_f$

$L = 0$

$P_f = P_\pi P_\pi (-1)^L = +1$

Parity not conserved.

Question 2 (b)

Feynman diagram with 3 pions can be separated at the gluon lines, therefore reaction is heavily suppress.

Question 3 (a)

$M = \frac{g_\omega^2}{8(M_\omega c)^2}[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)][\bar{u}(4)\gamma_\mu(1-\gamma^5)u(2)]$

$[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)]^\dagger = [u^\dagger(3)\gamma^0\gamma^\mu(1-\gamma^5)u(1)]^\dagger$

$= u^\dagger(1)(1-\gamma^5)^\dagger\gamma^{\mu\dagger}\gamma^{0\dagger}u(3)$

$= u^\dagger(1)\gamma^0\gamma^0(1-\gamma^5)\gamma^{\mu\dagger}\gamma^0 u(3)$

$= [\bar{u}(1)(1+\gamma^5)\gamma^\mu u(3)]$

$= [\bar{u}(1)\gamma^\mu(1-\gamma^5)u(3)]$

Hence

$$\sum_{s_1 s_2 s_3 s_3}|M|^2 = \left[\frac{g_\omega^2}{8(M_\omega c)^2}\right]^2[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)\bar{u}(1)\gamma^\nu(1-\gamma^5)u(3)][\bar{u}(4)\gamma_\mu(1 - \gamma^5)u(2)\bar{u}(2)\gamma_\nu(1-\gamma^5)u(4)]$$

Using $\sum_s u^{(s)}(p)\bar{u}^{(s)}(p) = \not{p} + mc$,

$$\sum_{s_1 s_2 s_3 s_3}|M|^2 = \left[\frac{g_\omega^2}{8(M_\omega c)^2}\right]^2 Tr[\gamma^\mu(1-\gamma^5)(\not{p}_1 + m_1 c)\gamma^\nu(1-\gamma^5)\not{p}_3]Tr[\gamma_\mu(1-\gamma^5)(\not{p}_2 + m_2 c)\gamma_\nu(1-\gamma^5)\not{p}_4]$$

$Tr[(\not{p}_1 + mc)\gamma^\nu(1-\gamma^5)\not{p}_3\gamma^\mu(1-\gamma^5)] = 2Tr[(\not{p}_1 + mc)\gamma^\nu(1-\gamma^5)\not{p}_3\gamma^\mu]$

$= 2Tr[(\not{p}_1 + mc)\gamma^\nu\not{p}_3\gamma^\mu - (\not{p}_1 + mc)\gamma^\nu\gamma^5\not{p}_3\gamma^\mu]$

$= 2[Tr(\not{p}_1\gamma^\nu\not{p}_3\gamma^\mu) - Tr(\not{p}_1\gamma^\nu\gamma^5\not{p}_3\gamma^\mu)]$

$= 2p_{1\lambda}p_{3\sigma}Tr(\gamma^\lambda\gamma^\nu\gamma^\sigma\gamma^\mu) - 2p_{1\lambda}p_{3\sigma}Tr(\gamma^\lambda\gamma^\nu\gamma^5\gamma^\sigma\gamma^\mu)$

$= 8[p_1^\nu p_3^\mu + p_1^\mu p_3^\nu - g^{\mu\nu}(\vec{p}_2\cdot\vec{p}_3) - i\epsilon^{\lambda\nu\sigma\mu}p_{1\lambda}p_{3\sigma}]$

Similarly, the second trace is:

$8[p_{2\nu}p_{4\mu} + p_{3\mu}p_{4\nu} - g_{\mu\nu}(\vec{p}_2\cdot\vec{p}_4) - i\epsilon_{\kappa\nu\tau\mu}p_2^\kappa p_4^\tau]$

Note that $p_1^\nu p_3^\mu + p_1^\mu p_3^\nu - g^{\mu\nu}(\vec{p}_2\cdot\vec{p}_3)$ is symmetric in $\mu \leftrightarrow \nu$ index whereas $\epsilon^{\lambda\nu\sigma\mu}$ is antisymmetric in $\mu \leftrightarrow \nu$, hence their product is zero.

$$\therefore \sum_{s_1 s_2 s_3 s_3}|M|^2 = 64\left[\frac{g_\omega^2}{8(M_\omega c)^2}\right]^2\{[p_1^\nu p_3^\mu + p_1^\mu p_3^\nu - g^{\mu\nu}(\vec{p}_2\cdot\vec{p}_3)][p_{2\nu}p_{4\mu} + p_{3\mu}p_{4\nu} - g_{\mu\nu}(\vec{p}_2\cdot\vec{p}_4)] + (-i\epsilon^{\lambda\nu\sigma\mu}p_{1\lambda}p_{3\sigma})(-i\epsilon_{\kappa\nu\tau\mu}p_2^\kappa p_4^\tau)\}$$

$[p_1^\nu p_3^\mu + p_1^\mu p_3^\nu - g^{\mu\nu}(\vec{p}_2\cdot\vec{p}_3)][p_{2\nu}p_{4\mu} + p_{3\mu}p_{4\nu} - g_{\mu\nu}(\vec{p}_2\cdot\vec{p}_4)]$

$= 2(\vec{p}_1\cdot\vec{p}_2)(\vec{p}_3\cdot\vec{p}_4) + 2(\vec{p}_1\cdot\vec{p}_4)(\vec{p}_2\cdot\vec{p}_3) - 2(\vec{p}_1\cdot\vec{p}_3)(\vec{p}_2\cdot\vec{p}_4) - 2(\vec{p}_2\cdot\vec{p}_4)(\vec{p}_1\cdot\vec{p}_3) + 4(\vec{p}_1\cdot\vec{p}_3)(\vec{p}_2\cdot\vec{p}_4)$

$= 2(\vec{p}_1\cdot\vec{p}_2)(\vec{p}_3\cdot\vec{p}_4) + 2(\vec{p}_1\cdot\vec{p}_4)(\vec{p}_2\cdot\vec{p}_3)$

$(-i)^2\epsilon^{\lambda\nu\sigma\mu}\epsilon_{\kappa\nu\tau\mu}p_{1\lambda}p_{3\sigma}p_2^\kappa p_4^\tau = 2[(\vec{p}_1\cdot\vec{p}_2)(\vec{p}_3\cdot\vec{p}_4) - (\vec{p}_1\cdot\vec{p}_4)(\vec{p}_2\cdot\vec{p}_3)]$

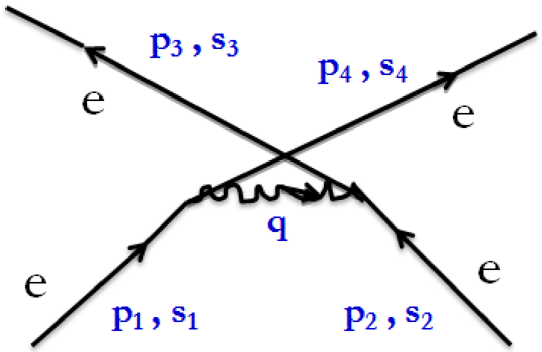
Adding both, we have $\rightarrow 4(\vec{p}_1\cdot\vec{p}_2)(\vec{p}_3\cdot\vec{p}_4)$.

$$\therefore \sum_{s_1 s_2 s_3 s_3} |M|^2 = 4\left[\frac{g_\omega}{M_\omega c}\right]^4 (\vec{p}_1 \cdot \vec{p}_2)(\vec{p}_3 \cdot \vec{p}_4)$$

Now we sum over final e^- spins but average over initial μ^- spins (2 possible spins), hence factor of $\frac{1}{2}$.

$$\therefore \langle |M|^2 \rangle = 2\left[\frac{g_\omega}{M_\omega c}\right]^4 (\vec{p}_1 \cdot \vec{p}_2)(\vec{p}_3 \cdot \vec{p}_4)$$

Question 3 (b)



$$M = -\frac{g_e^2}{(p_1 - p_3)^2}[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_4)}(p_4)\gamma_\mu u^{(s_2)}(p_2)] + \frac{g_e^2}{(p_1 - p_4)^2}[\bar{u}^{(s_4)}(p_4)\gamma^\mu u^{(s_1)}(p_1)][\bar{u}^{(s_3)}(p_3)\gamma_\mu u^{(s_2)}(p_2)]$$

Question 4 (a)
Question 4 (b)

Question 4 (c)

$$(\gamma^\mu p_\mu - mc)\, u(p) = 0$$

$$u^{(1)} = \begin{pmatrix} 1 \\ 0 \\ \frac{cP_z}{E+mc^2} \\ \frac{c(P_x + iP_y)}{E+mc^2} \end{pmatrix} \qquad u^{(2)} = \begin{pmatrix} 0 \\ 1 \\ \frac{c(P_x - iP_y)}{E+mc^2} \\ \frac{-cP_z}{E+mc^2} \end{pmatrix}$$

$$\gamma^5 u(p) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

$$= \begin{pmatrix} u_B \\ u_A \end{pmatrix}$$

$$\begin{pmatrix} \frac{c(\vec{p}\cdot\sigma)}{E+mc^2} & 0 \\ 0 & \frac{c(p\cdot\sigma)}{E-mc^2} \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

$$= \begin{pmatrix} \frac{c(\vec{p}\cdot\vec{\sigma})}{E+mc^2} u_A \\ \frac{c(p\cdot\sigma)}{E+mc^2} u_B \end{pmatrix}$$

$$\vec{p}\cdot\vec{\sigma}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} P_z & P_x - iP_y \\ P_x + iP_y & -P_z \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} P_z \\ P_x + iP_y \end{pmatrix}$$

$$\vec{p}\cdot\vec{\sigma}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} P_x - iP_y \\ -P_z \end{pmatrix}$$

$$\frac{c\vec{p}\cdot\sigma}{E-mc^2}\begin{pmatrix} P_z \\ P_x + iP_y \end{pmatrix} = \frac{c}{E-mc^2}\begin{pmatrix} P_z & P_x - iP_y \\ P_x + iP_y & -P_z \end{pmatrix}\begin{pmatrix} P_z \\ P_x + iP_y \end{pmatrix}$$

$$= \frac{c}{E-mc^2}\begin{pmatrix} |P|^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\frac{c\vec{P}\cdot\sigma}{E-mc^2}\begin{pmatrix} P_x - iP_y \\ -P_z \end{pmatrix} = \frac{c}{E-mc^2}\begin{pmatrix} 0 \\ |P|^2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Solutions provided by:

Prof. Teo Kien Boon (Questions **3a-b**)

Bong Kok Wei (Questions **1, 2a(i),(ii),(iv), 2b, 4c**)