This document lacks answers for certain questions. Would you like to help us complete them? If yes, Please send your suggested answers to nus.physoc@gmail.com. Thanks! ☺

Question 1 (a)

Use conservation of 4-momentum: $P_A + P_B = P_C + P_D$

Rearranging to get: $P_A - P_C = P_D - P_B$

Taking the invariant dot product squared on both sides, and noting that A and C are massless:

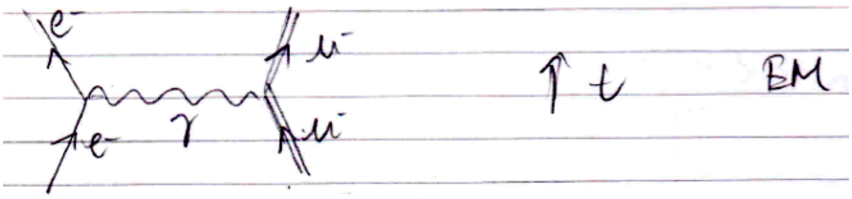
$$-2\left(\frac{E_A E_C}{c^2} - \vec{p}_A \cdot \vec{p}_C\right) = m_D^2 c^2 + m_B^2 c^2 - 2E_D m_B$$

Since $\vec{p}_A \cdot \vec{p}_C = \dfrac{E_A E_C \cos\theta}{c^2}$,

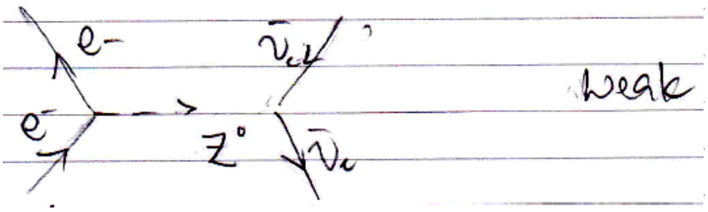
and $E_D = E_A + m_B c^2 - E_C$ from energy conservation, we get, after simplifying:

$$\cos\theta = \frac{2E_A E_C - 2m_B c^2 (E_A - E_C) + (m_D^2 - m_B^2)c^4}{2E_A E_C}$$

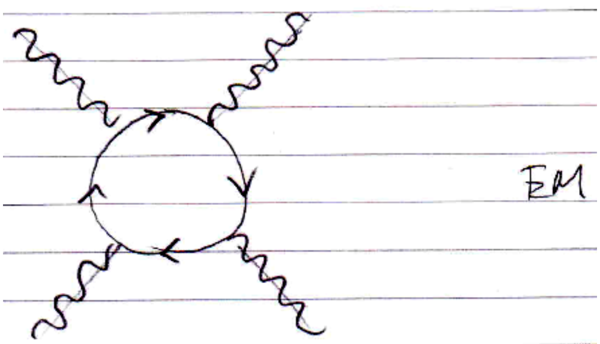
Question 1 (b) (i)



Question 1 (b) (ii)

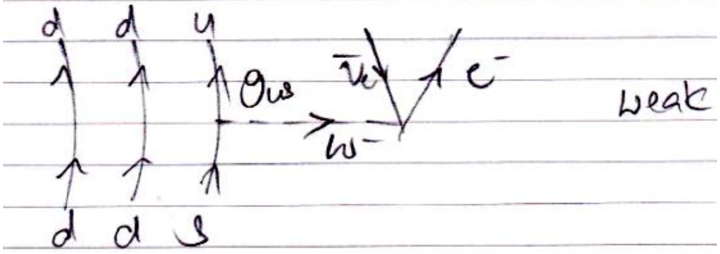


Question 1 (b) (iii)



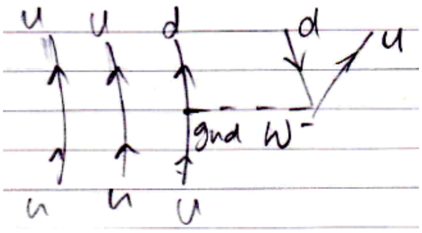
Question 1 (b) (iv)

$dds \to udd + e^- + \bar{\nu}_e$



Question 1 (b) (v)

$uuu \to uud + u\bar{d}$



Question 1 (c)

SU(3) flavour is not an exact symmetric while SU(3) colour is an exact symmetry where the interchange of particle will not affect the physics.

Question 2 (a)

$K^- + p = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = |1\,1\rangle$

$\bar{K}^0 + p = |\tfrac{1}{2}\,\tfrac{1}{2}\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{2}}\,|1\,0\rangle - \sqrt{\tfrac{1}{2}}\,|0\,0\rangle$

(i) : (ii) : (iii) : (iv)

$\Sigma^0\ \pi^0$: $|1\,0\rangle|1\,0\rangle = \sqrt{\tfrac{2}{3}}\,|2\,0\rangle - \sqrt{\tfrac{1}{3}}\,|0\,0\rangle$

$\Sigma^+\ \pi^-$: $|1\,1\rangle|1\,-1\rangle = \sqrt{\tfrac{1}{6}}\,|2\,0\rangle + \sqrt{\tfrac{1}{2}}\,|1\,0\rangle + \sqrt{\tfrac{1}{3}}\,|0\,0\rangle$

$\Sigma^+\ \pi^0$: $|1\,1\rangle|1\,0\rangle = \sqrt{\tfrac{1}{2}}\,|2\,1\rangle + \sqrt{\tfrac{1}{2}}\,|1\,1\rangle$

$\Sigma^0 + \pi^+$: $|1\,0\rangle|1\,1\rangle = \sqrt{\tfrac{1}{2}}\,|2\,1\rangle - \sqrt{\tfrac{1}{2}}\,|1\,1\rangle$

(i) (ii) (ii) (iv)

$0 : \tfrac{1}{2}M_1 : 0 : 0$

Question 2 (b)

Conservation of angular momentum requires: $J_i = J_f$

$J_i = J_A = 1$; $J_f = L_{BC}$ (since $J_B = J_C = 0$)

Hence $L_{BC} = 1$.

Conservation of parity requires: $P_A = P_{BC} = P_B \cdot P_C \cdot (-1)^{L_{BC}}$

Since $L_{BC} = 1$, this gives us the selection rule: $P_A = -P_B \cdot P_C$.

So allowed decays are:

$1^+ \to 0^+ + 0^-$

$1^+ \to 0^- + 0^+$

$1^- \to 0^+ + 0^+$

$1^- \to 0^- + 0^-$

Forbidden decays are:

$1^+ \to 0^+ + 0^+$

$1^+ \to 0^- + 0^-$

$1^- \to 0^+ + 0^-$

$1^- \to 0^- + 0^+$

Question 2 (c)

In weak interactions, all interacting neutrinos are found to be left-handed and all interacting antineutrinos right-handed. Hence we can label the handedness of the 2 given reactions as follows:

(1) $\mu^- \to e^- + \bar{\nu}_e(R) + \bar{\nu}_\mu(L)$

(2) $\mu^+ \to e^+ + \nu_e(L) + \bar{\nu}_\mu(R)$

where (L) and (R) indicates the handedness respectively.

However, C converts neutrinos into antineutrinos and vice versa, without affecting the handedness. Hence applying C to reaction (1) and (2) yields:

C-transformed (1) : $\mu^+ \to e^+ + \nu_e(R) + \bar{\nu}_\mu(L)$

C-transformed (2) : $\mu^- \to e^- + \bar{\nu}_e(L) + \bar{\nu}_\mu(R)$

which each contains unphysical neutrinos and antineutrinos with the incorrect handedness. Hence reactions (1) and (2) provide evidence for the non-invariance of C.

However, the combined CP operation changes left-handed neutrinos into right-handed antineutrinos and vice versa, both of which participate in weak interactions; and hence CP converts reaction (1) into reaction (2) and vice versa. So the existence of the two reactions is consistent with the invariance of CP.

Question 3 (a)

Decay of $D^+(c\bar{d})$ meson $\begin{matrix} \nearrow K^-\pi^+\pi^+ \\ \searrow K^+\pi^-\pi^+ \end{matrix}$

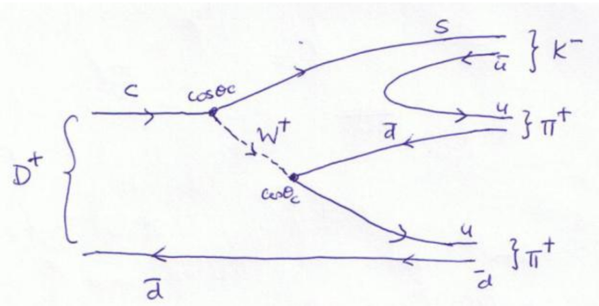


Diagram 1: $D^+ \to K^-\pi^+\pi^+$

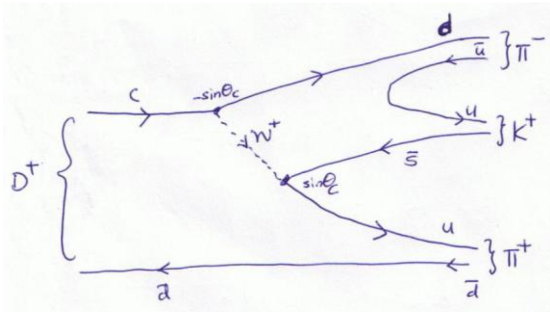


Diagram 2: $D^+ \to K^+\pi^-\pi^+$

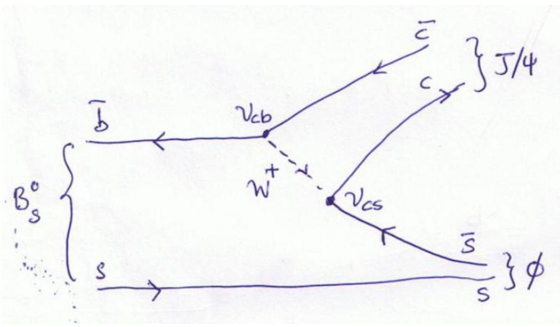
By Cabibbo theory, the quark W^+ coupling has a factor of $\cos\theta_c$ if the 2 quark flavours involved come from the same generation (i.e. c with s, u with d) but a "suppressed" factor of $\sin\theta_c$ if the 2 quark flavours come from different generations (i.e. c with d, u with s). Hence from diagram 1, $D^+ \to K^-\pi^+\pi^+$ decay mode, the decay rate will be proportional to $\cos^4\theta_c$ whereas from diagram 2, $D^+ \to K^+\pi^-\pi^+$ decay mode, the decay rate will be proportional to $\sin^4\theta_c$.

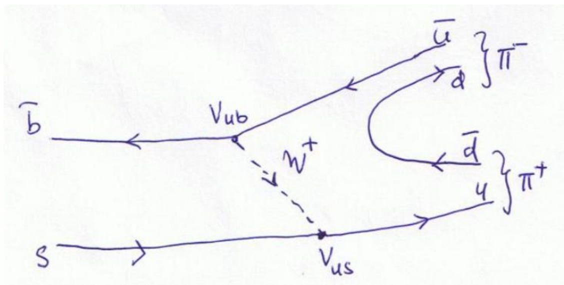
$$\therefore \text{ Rate of } \frac{D^+ \to K^-\pi^+\pi^+}{D^+ \to K^+\pi^-\pi^+} \approx \left(\frac{\cos\theta_c}{\sin\theta_c}\right)^4 \approx 20^2 = 400 \text{ times!}$$

Question 3 (b)

(i) $B_s^0(\bar{b}s) \to {}^J\!/_\psi(c\bar{c}) + \phi(s\bar{s})$

(ii) $B_s^0(\bar{b}s) \to \pi^+\pi^-$





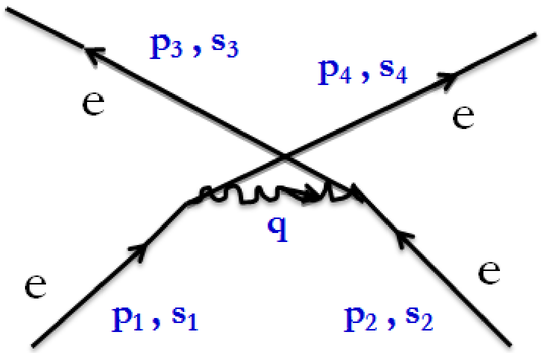
From CKM matrix, $\begin{matrix} V_{cb} = 0.0410 \\ V_{cs} = 0.9735 \end{matrix}$ whereas $\begin{matrix} V_{ub} = 0.00347 \\ V_{us} = 0.2253 \end{matrix}$

Since rate of $B_s^0(\bar{b}s) \to {}^J\!/_\psi(c\bar{c}) + \phi(s\bar{s})$ will incur factors of $|V_{cb}|^2|V_{cs}|^2$ while rate of $B_s^0(\bar{b}s) \to \pi^+\pi^-$ will incur factors of $|V_{ub}|^2|V_{us}|^2$, the values of the $|V_{ij}|^2$ tell us that rate of $B_s^0(\bar{b}s) \to {}^J\!/_\psi(c\bar{c}) + \phi(s\bar{s}) \gg B_s^0(\bar{b}s) \to \pi^+\pi^-$.

Question 3 (c)

Question 4 (a)

Question 4 (b)



$$M = -\frac{g_e^2}{(p_1-p_3)^2}\left[\bar{u}^{(s_3)}(p_3)\gamma^\mu u^{(s_1)}(p_1)\right]\left[\bar{u}^{(s_4)}(p_4)\gamma_\mu u^{(s_2)}(p_2)\right] + \frac{g_e^2}{(p_1-p_4)^2}\left[\bar{u}^{(s_4)}(p_4)\gamma^\mu u^{(s_1)}(p_1)\right]\left[\bar{u}^{(s_3)}(p_3)\gamma_\mu u^{(s_2)}(p_2)\right]$$

Question 4 (c)

Solutions provided by:

Prof. Teo Kien Boon (Questions **1a, 2b-c, 3a-b, 4b**)

Bong Kok Wei (Questions **1b-2a**)