NATIONAL UNIVERSITY OF SINGAPORE

PC4245 PARTICLE PHYSICS

(Semester II: AY 2011-12)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FOUR (4)** questions and comprises **SIX (6)** printed pages.
2. Answer **ANY THREE (3)** questions.
3. Answers to the questions are to be written in the answer books.
4. This is a **CLOSED BOOK** examination.
5. **One Help Sheet (A4 size, both sides) is allowed** for this examination.
6. The Clebsch-Gordan coefficient table is attached as the last printed page.

Question 1 (6+15+4 = 25 marks)

(a) Consider a two-body scattering event, $A + B \rightarrow C + D$, where particles A and C are both massless. Prove that in the laboratory frame (B at rest), the angle θ between the beam directions of particles A and C is given by:

$$\cos(\theta) = f\,(E_A, E_C, m_B, m_D)$$

and derive this function f explicitly.

(b) Each of the following reactions may occur via a QED process, a QCD process, a weak interaction process, or via more than one of these processes. For each reaction, draw the lowest order Feynman diagram(s), clearly labeling each lepton, quark and boson. In each case, state also which fundamental forces are involved.

(i) $e^- + e^+ \rightarrow \mu^- + \mu^+$

(ii) $e^- + \bar{\nu}_e \rightarrow e^- + \bar{\nu}_e$

(iii) $\gamma + \gamma \rightarrow \gamma + \gamma$

(iv) $\Sigma^-(dds) \rightarrow n + e^- + \bar{\nu}_e$

(v) $\Delta^{++} \rightarrow p + \pi^+$

(c) Discuss, in the context of the strong interactions, the differences between flavour SU(3) symmetry and colour SU(3) symmetry.

Question 2 (10+9+6 = 25 marks)

(a) The two kaons $(K^-, \overline{K}^0)$ form a doublet with isospin-$\frac{1}{2}$ while the three sigma baryons $(\Sigma^-, \Sigma^0, \Sigma^+)$ form a triplet with isospin-1 just like the pions.

Find the ratio of the cross sections for the following reactions, assuming that the $I = 1$ channel dominates:

(i) $K^- + p \rightarrow \Sigma^0 + \pi^0$

(ii) $K^- + p \rightarrow \Sigma^+ + \pi^-$

(iii) $\overline{K}^0 + p \rightarrow \Sigma^+ + \pi^0$

(iv) $\overline{K}^0 + p \rightarrow \Sigma^0 + \pi^+$

(b) Consider the strong decay of a spin-1 meson A (i.e. J_A=1) into two spinless mesons B and C (i.e. $J_B = J_C = 0$).

Derive selection rules for the possible J^P assignments of A, B and C.

For example, is $1^+ \rightarrow 0^+ + 0^+$ possible? In other words, derive all selection rules showing which combinations are allowed and which are forbidden.

(Hint: invoke appropriate conservation laws.)

(c) Decay of the negative muon is very similar to that of the positive muon, except that all particles are swapped for their antiparticles:

$$\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$$

Discuss these reactions in the context of C invariance and CP invariance.

Question 3 (8+12+5 = 25 marks)

(a) Using Cabibbo theory, explain, with the help of diagrams, why the D^+ meson $(c\bar{d})$ has a much higher preference for the decay mode into $K^-\pi^+\pi^+$ compared with the decay mode into $K^+\pi^-\pi^+$.

(Note that the quark contents of the charged kaons are respectively $K^-(s\bar{u})$ and $K^+(u\bar{s})$.)

(b) The Cabibbo-Kobayashi-Maskawa (CKM) matrix relates the weak interaction quark states (d', s', b') to the physical quark states (d, s, b) as follows:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The experimental values of the CKM matrix are given below:

$$|V_{ij}| = \begin{pmatrix} 0.97428 & 0.2253 & 0.00347 \\ 0.2252 & 0.97345 & 0.0410 \\ 0.00862 & 0.0403 & 0.999152 \end{pmatrix}$$

Draw the Feynman diagrams for the following two decay modes of the meson $B_s^0(\bar{b}s)$, and use the CKM matrix to explain why the decay mode of this B-meson to the two-pion final state is much more suppressed.

(i) $B_s^0(\bar{b}s) \rightarrow J/\varphi\ (c\bar{c}) + \phi\,(s\bar{s})$

(ii) $B_s^0(\bar{b}s) \rightarrow \pi^+ + \pi^-$

(c) Briefly discuss the differences between global gauge invariance and local gauge invariance, and their roles in the Standard Model description of interactions.

Question 4 (16+6+3 = 25 marks)

(a) Draw the lowest-order Feynman diagram for the following process:

$$e^- + e^+ \rightarrow \mu^- + \mu^+$$

Using the Feynman rules for quantum electrodynamics, obtain the scattering amplitude M for the above process.

Using the Casimir trick and the appropriate trace theorems, prove that the spin-averaged quantity <|M|^2>, in the high energy limit (i.e. neglecting all particle masses), is given as follows:

$$\langle |M|^2 \rangle = \frac{8g_e^4}{(p_1+p_2)^4}\left[(p_1\cdot p_4)(p_2\cdot p_3)+(p_1\cdot p_3)(p_2\cdot p_4)\right]$$

Note: The following formulas can be used without proof:

- $\sum_s u^{(s)}(p)\,\bar{u}^{(s)}(p) = \not{p} + mc$
- $\sum_s v^{(s)}(p)\,\bar{v}^{(s)}(p) = \not{p} - mc$
- $Tr\left[\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma\right] = 4\left[g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda}\right]$
- $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\,g^{\mu\nu}$; $\gamma^0\gamma^{\mu+}\gamma^0 = \gamma^\mu$

(b) Draw the two lowest-order Feynman diagrams for electron-electron scattering:

$$e^- + e^- \rightarrow e^- + e^-$$

Using the Feynman rules for quantum electrodynamics, obtain the scattering amplitude M for the above process.

(c) Draw any six fourth-order (four-vertex) Feynman diagrams for the following process:

$$e^- + e^+ \rightarrow \mu^- + \mu^+$$

(TKB)

- END OF PAPER -

36. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

		J J … M M …
m_1 m_2		
m_1 m_2 ⋮ ⋮		Coefficients

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2×1/2

m_1	m_2	1 +1
+1/2	+1/2	1

m_1	m_2	1 0	0 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	1 −1
−1/2	−1/2	1

1×1/2

m_1	m_2	3/2 +3/2
+1	+1/2	1

m_1	m_2	3/2 +1/2	1/2 +1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	3/2 −1/2	1/2 −1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	3/2 −3/2
−1	−1/2	1

2×1

m_1	m_2	3 +3
+2	+1	1

m_1	m_2	3 +2	2 +2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	3 +1	2 +1	1 +1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	3 0	2 0	1 0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	3 −1	2 −1	1 −1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	3 −2	2 −2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	3 −3
−2	−1	1

1×1

m_1	m_2	2 +2
+1	+1	1

m_1	m_2	2 +1	1 +1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	2 0	1 0	0 0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	2 −1	1 −1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	2 −2
−1	−1	1

2×1/2

m_1	m_2	5/2 +5/2
+2	+1/2	1

m_1	m_2	5/2 +3/2	3/2 +3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	5/2 +1/2	3/2 +1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	5/2 −1/2	3/2 −1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	5/2 −3/2	3/2 −3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	5/2 −5/2
−2	−1/2	1

3/2×1/2

m_1	m_2	2 +2
+3/2	+1/2	1

m_1	m_2	2 +1	1 +1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	2 0	1 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	2 −1	1 −1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	2 −2
−3/2	−1/2	1

3/2×1

m_1	m_2	5/2 +5/2
+3/2	+1	1

m_1	m_2	5/2 +3/2	3/2 +3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	5/2 +1/2	3/2 +1/2	1/2 +1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	5/2 −3/2	3/2 −3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	5/2 −5/2
−3/2	−1	1

3/2×3/2

m_1	m_2	3 +3
+3/2	+3/2	1

m_1	m_2	3 +2	2 +2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	3 +1	2 +1	1 +1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	3 0	2 0	1 0	0 0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	3 −1	2 −1	1 −1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	3 −2	2 −2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	3 −3
−3/2	−3/2	1

2×3/2

m_1	m_2	7/2 +7/2
+2	+3/2	1

m_1	m_2	7/2 +5/2	5/2 +5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	7/2 +3/2	5/2 +3/2	3/2 +3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	7/2 +1/2	5/2 +1/2	3/2 +1/2	1/2 +1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	7/2 −1/2	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	7/2 −3/2	5/2 −3/2	3/2 −3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	7/2 −5/2	5/2 −5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	7/2 −7/2
−2	−3/2	1

2×2

m_1	m_2	4 +4
+2	+2	1

m_1	m_2	4 +3	3 +3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	4 +2	3 +2	2 +2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	4 +1	3 +1	2 +1	1 +1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	4 0	3 0	2 0	1 0	0 0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	4 −1	3 −1	2 −1	1 −1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	4 −2	3 −2	2 −2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	4 −3	3 −3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	4 −4
−2	−2	1

d functions

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 36.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974).