NATIONAL UNIVERSITY OF SINGAPORE

PC4246 Quantum Optics

(Semester I: AY 2014-15)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1 This assessment paper contains 3 questions and comprises 5 printed pages (including this page).

2 Students are required to answer ALL questions.

3 The answers are to be written with ink pen only (no pencil).

4 This is a CLOSED BOOK examination.

5 One help sheet (A4 size both sides) is allowed for this examination

6 Students should write the answers for each question on a new page.

7 The total mark is 60.

8 Mark for each question is shown in square bracket.

9 Programmable calculators are NOT allowed.

Q1. Figure (1) describes a Mach-Zehnder interferometer setup with a balanced homodyne detection at its output. Based on this, answer the following questions: [**marks = 25**]

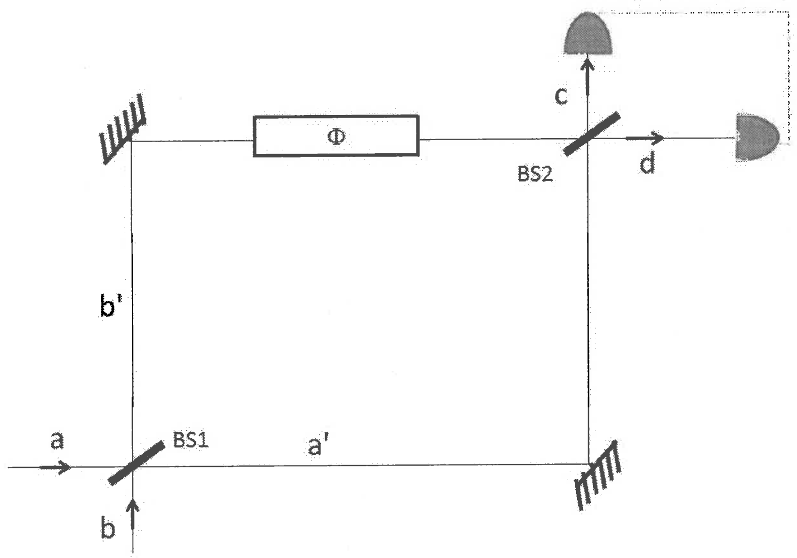

Figure 1: Mach-Zehnder interferometer setup with local oscillator input b and quantum state input a. The upper arm contains a phase shifter. The two beam splitters (BS1 and BS2) are 50:50 beam splitter.

A. Derive the output homodyne operator in terms of the input mode creation operators (HINT: homodyne observable is the difference in photon counts of the detectors).

B. Consider that the local oscillator mode is a large amplitude coherent state. The input quantum state a is a single mode vacuum state. Show that the photon number in the local oscillator mode can be measured by this setup. Keep in mind that $\langle n \rangle = |\beta_l|^2$ where $|\beta_l|$ is the coherent state amplitude.

C. Under what condition would this setup behave as a balanced homodyne detection setup (HINT: for balanced homodyning only the interference term remains).

D. Considering the input quantum state a as single mode vacuum and b as large amplitude coherent state, derive that the variance of the balanced homodyne detection measurement.

E. Question no. (D) gives the error in the photon number measurement. From the relation of photon number and phase, show that the phase error of the measurement is inversely proportional to the square root of photon number in the local oscillator mode.

F. If now, we replace the input signal a by a squeezed **vacuum** state $|0, \xi\rangle$ where squeezing parameter $\xi = r \exp i\theta$, show that the phase noise is exponentially reduced. As the local oscillator has a large amplitude you may neglect $\sinh^2 r$ terms.

Some operator expectation values which you may need

For a coherent squeezed state $|\alpha, \xi\rangle$ (where $\xi = r \exp i\theta$), the following expectation values hold:

$$\langle a \rangle = \alpha \cosh r - \alpha^* \exp i\theta \sinh r \tag{1}$$

$$\langle a^2 \rangle = \langle a^{\dagger 2} \rangle^*$$

$$= \alpha^2 \cosh^2 r + (\alpha^*)^2 \exp 2i\theta \sinh^2 r - 2|\alpha|^2 \exp i\theta \sinh r \cosh r - \exp i\theta \cosh r \sinh r \tag{2}$$

$$\langle a^\dagger a \rangle = |\alpha|^2(\cosh^2 r + \sinh^2 r) - (\alpha^*)^2 \exp i\theta \sinh r \cosh r - \alpha^2 \exp -i\theta \sinh r \cosh r + \sinh^2 r \tag{3}$$

Q2. Consider the following three operators

$$\hat{A}_1 = \frac{1}{2}(\hat{a}^{\dagger 2} + \hat{a}^2)$$
$$\hat{A}_2 = \frac{1}{2i}(\hat{a}^{\dagger 2} - \hat{a}^2)$$
$$\hat{A}_3 = \frac{1}{2}(\hat{a}^{\dagger}\hat{a} + \frac{1}{2})$$

The first two Hermitian operators are known as the quadrature of amplitude square operators. Based on this, answer the following questions: [**marks = 20**]

A. "The above three operators form a close set under commutation" (meaning: commutation of any two is the third). As an example, verify that the commutation of $\hat{A}_1$ and $\hat{A}_2$ is indeed $\hat{A}_3$.

B. Derive the uncertainty relation between $\hat{A}_1$ and $\hat{A}_2$.

C. Show that the coherent state is the minimum uncertainty state for the amplitude square quadrature.

D. What would be the condition for squeezing?

Q3. Figure (2) shows the diagram for two photon transition like the famous 1s to 2s transition in Hydrogen. For such a process the Hamiltonian is a modified Jaynes-Cummings Hamiltonian given by $H = \hbar\eta(\hat{a}^2\hat{\sigma}_+ + \hat{a}^{\dagger 2}\hat{\sigma}_-)$ (for simplicity the Stark shift contribution has been ignored). Based on this, answer the following questions: [**marks**= 15]

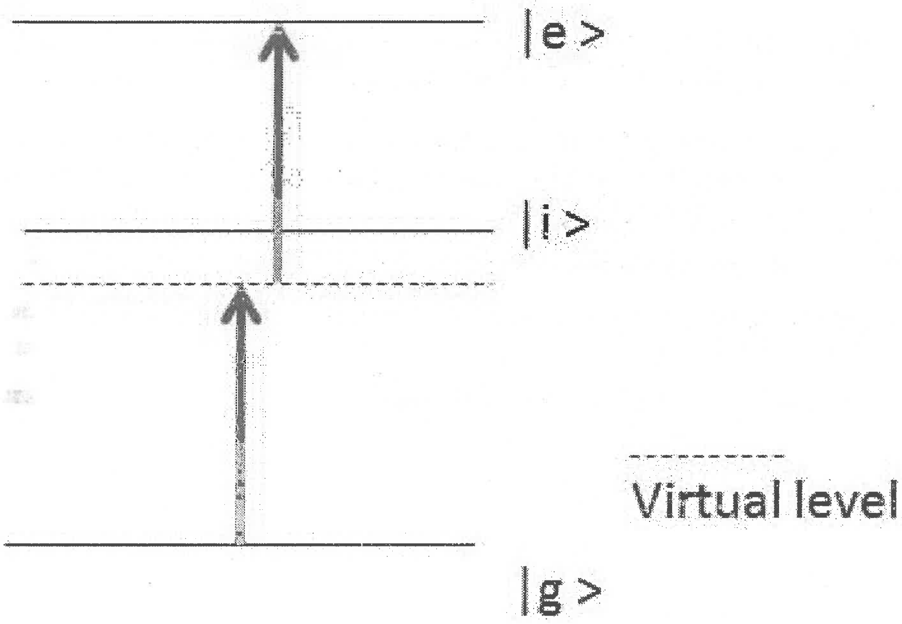

Figure 2: Two photon transition: The level $|e\rangle$ and $|g\rangle$ are of same parity while the intermediate level $|i\rangle$ is of opposite parity. The dashed line denotes virtual levels which are detuned from the level $|i\rangle$.

A. Derive the dressed states for two-photon transitions.

B. Obtain the time evolution of the system starting from the atom in ground state and the field in a number state.

D. Comment on the collapse and revival behaviour of the system.

END OF PAPER

[MM]