NATIONAL UNIVERSITY OF SINGAPORE

PC4246 Quantum Optics

(Semester I: AY 2015-16)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1 Please write your student number only on the answer book. Do not write your name.

2 This assessment paper contains 3 questions and comprises 3 printed pages (including this page).

3 Students are required to answer ALL questions.

4 The answers are to be written with ink pen only (no pencil).

5 This is a CLOSED BOOK examination.

6 Students should write the answers for each question on a new page.

7 The total mark is 40.

8 Mark for each question is shown in square bracket.

9 Programmable calculators are NOT allowed.

Q1. A linearly polarized mono-mode laser light propagating in vacuum is incident on a quarter wave plate as shown in the Figure 1. The laser has an output power of 30 mW and a beam waist diameter of 4 mm. Neglect any divergence of the beam. Based on this information answer the following questions: [15 marks]

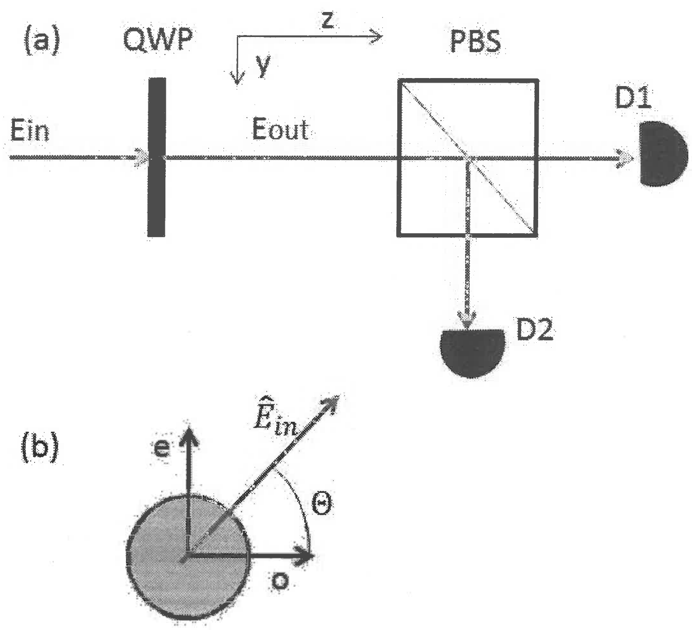

Figure 1: (a) The optical circuit with input laser beam (E_{in}) passing through a Quarter Wave Plate (QWP) generates output beam (E_{out}). The output propagates through a Polarizing Beam Splitter (PBS) and finally detected by two detectors $D1$ and $D2$. (b) The orientation of the principle axes (o and e) of a QWP is shown with respect to the incoming beam polarization ($\hat{E_{in}}$).

A. Write down the expression for the classical electric field of the laser at the input of the optical system.

B. Starting from the working principle of a general wave plate (birefringent material), calculate the angle θ of the quarter wave plate for which the laser beam coming out of the quarter wave plate will have pure circular polarization (either right or left).

C. Now consider the quantum state of the laser to be single mode coherent state. Write down the complete electric field operator in terms of plane wave components.

D. In continuation of the above question [C], derive the expectation value of the electric field and show that it is similar to a classical field.

E. Calculate the output intensities (with proper units) as observed by the detectors D1 and D2 placed at the output of the polarizing beam splitter (PBS).

Q2. Consider any mono-mode arbitrary pure quantum state of light $|\psi\rangle$. Absorption of a single photon produces a state $|\tilde{\psi}\rangle = a|\psi\rangle$. Remember $\bar{n} = \langle\psi|\,\hat{n}\,|\psi\rangle$ and $\bar{\bar{n}} = \langle\tilde{\psi}|\,\hat{n}\,|\tilde{\psi}\rangle$ respectively. [15 marks]

A. What is the mean photon number for the state $|\psi\rangle$? [HINT: Use the Fock state basis to express any arbitrary state]

B. Normalize the state $|\tilde{\psi}\rangle$.

C. Derive the expression for the mean photon number in the normalized state of $|\tilde{\psi}\rangle$ as a function of ψ. [HINT: Use commutator relation $[a, a^\dagger] = 1$]

D. From the result of question [C] infer whether $\bar{\bar{n}} = \bar{n} - 1$. Comment on the outcome of the result.

E. Consider a pure equal superposition mono-mode state of vacuum and 8-photon Fock state $|\psi\rangle = 1/\sqrt{2}(|0\rangle + |8\rangle$. Calculate the mean photon number in the states $|\psi\rangle$ and $|\tilde{\psi}\rangle = a|\psi\rangle$. Compare this result with the previous result obtained in question [D].

Q3. The following questions are based on first and second order coherence of light field: [10 marks]

A. Prove that thermal state of light has second order coherence of 2.

B. Derive the first and second order coherences for the following two states:

a. $|\psi\rangle = C_0\,|0\rangle + C_1\,|1\rangle$
b. $\rho = |C_0|^2\,|0\rangle + |C_1|^2\,|1\rangle$

where $|C_0|^2 + |C_1|^2 = 1$.

C. Comment on the result obtained in [B].

END OF PAPER

[MM]