NATIONAL UNIVERSITY OF SINGAPORE

PC4246 Quantum Optics

(Semester I: AY 2016-17)

Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1 Please write your student number only on the answer book. Do not write your name.

2 This assessment paper contains 3 questions and comprises 3 printed pages (including this page).

3 Students are required to answer ALL questions.

4 The answers are to be written with ink pen only (no pencil).

5 This is a CLOSED BOOK examination.

6 Students should write the answers for each question on a new page.

7 The total mark is 40.

8 Mark for each question is shown in square bracket.

9 Programmable calculators are NOT allowed.

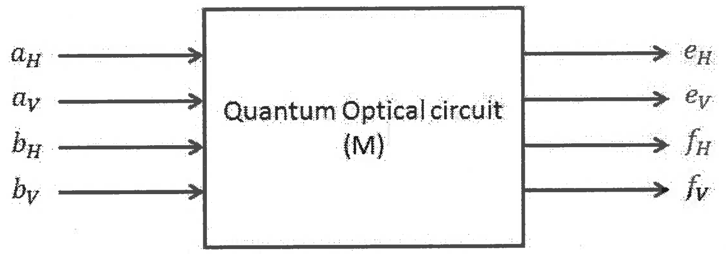
PART A [4×5]

In this section please choose the most appropriate answer(s) and write it in your answer booklet. **You need to show the work out wherever necessary to justify your choice.**

Q1.1 In case of electromagnetic field in a state $|\psi\rangle$ is incident on a single photon detector like a photo multiplier tube the detected signal is proportional to:

(A) $\langle \hat{E}^{\dagger}\hat{E}\rangle$ (B) $\langle \hat{a} + \hat{a}^{\dagger}\rangle$ (C) $\langle \hat{E}^{(-)}\hat{E}^{(+)}\rangle$ (D) None of the above

Q1.2 A black box with four input and out modes is sold to you as a generic quantum optical circuit:



The transformation matrix M (from input to output) corresponding to the box is found to be:

$$M = -\frac{1}{2}\begin{pmatrix} 1 & x & i & ix \\ x & 1 & ix & i \\ i & ix & 1 & x \\ ix & i & x & 1 \end{pmatrix}. \quad (1)$$

where $x = e^{i\theta}$ and θ is real.

With no further information, which is the following statement(s) can you infer from the box:

(A) It is a genuine quantum optical circuit
(B) It is not a quantum optical circuit
(C) Nothing can be definitively said about the nature of the box

Q1.3 A monomode two photon state $|2\rangle_V$ (polarization modes only) where V denotes the vertical polarization mode, is incident on a photo detector. If the detector is an *ideal intensity detector* but can only detect right circular polarized photons, what fraction of the input intensity will be detected by the detector?

(A) 1/2 (B) 0 (C) 1/4 (D) None of the above

Q1.4 The eigenvalue of creation operator $\hat{a}^\dagger$ is expressed in terms of complex number α as:

(A) α^* (B) $\alpha^\dagger$ (C) α (D) None of the above

PART B [12+8]

Q2. The quadrature operators are very useful in quantum optics and they are given by

$$\hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger) \tag{2}$$

$$\hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger). \tag{3}$$

Consider a monomode superposition state in number basis $|\psi\rangle = c_0 |0\rangle + c_1 |1\rangle$, where $|c_0|^2 + |c_1|^2 = 1$. As we know, squeezing occurs when the variance $(\langle x^2\rangle - \langle x\rangle^2)$ of the quadrature in a given state is below the variance in vacuum.

Q2.1 Calculate the variance of quadratures $\hat{X}_1$ and $\hat{X}_2$ in state $|\psi\rangle$ as a function of c_0 and c_1.

Q2.2 Express c_1 in terms of c_0 and a complex phase ϕ as $c_1 = (\sqrt{1 - |c_0|^2})e^{i\phi}$ and $c_0^2 = |c_0|^2$. Show for any of the quadratures (X_1 or X_2) squeezing occurs for certain values of parameter c_0 and ϕ. [reminder: the value in vacuum for the quadratures is 0.25].

Q2.3 How does the previous result affect the uncertainty relationship between the quadratures?

Q3. Coherence of a state is partially characterized by the first $(E^{(-)}E^{(+)})$ and second order $(E^{(-)}E^{(-)}E^{(+)}E^{(+)})$ correlations $g^{(1)}$ and $g^{(2)}$. In the following these correlations are verified on certain states:

Q3.1 Calculate the second order correlation for a Schrödinger cat state which is a superposition state of two coherent states with large $|\alpha|$ as given by $|\psi\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle)$. [Hint: use $\langle\alpha| - \alpha\rangle = 0$ for large $|\alpha|$.]

Q3.2 Calculate the second order correlation for a mixture of coherent states as given by $\rho = \frac{1}{2}(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)$.

Q3.3 Explain why the results are same or different in the above two questions.

END OF PAPER

[MM]