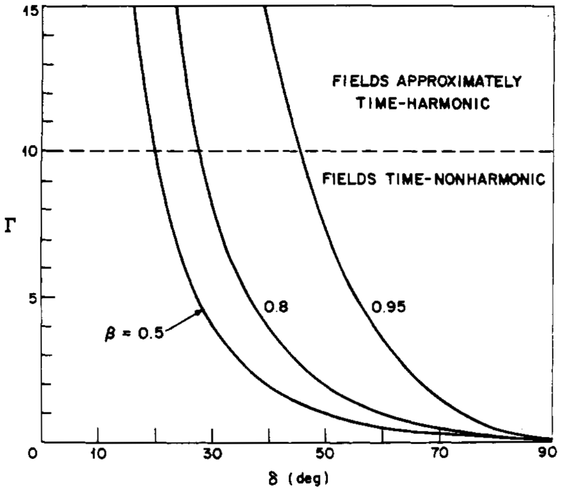

Fig. 2. The function $\Gamma = \gamma^2 \cot^2 \delta$ plotted versus the motion angle δ for various velocities.

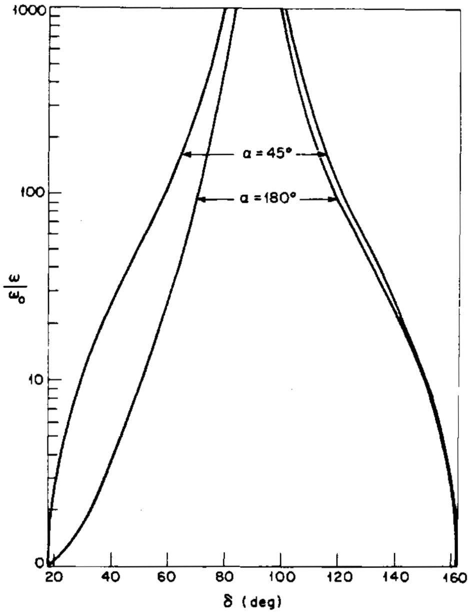

Fig. 3. Doppler shift versus δ for various incidence angles under the constraint $\gamma^2 \cot^2 \delta = 10$.

and if β is not too close to zero,[5] we may write

$$\rho' \sim \gamma(x \cos \delta - v_0 t). \tag{12}$$

The inequality (11) is satisfied exactly for motion along the x direction. For motion in an arbitrary direction, it can be satisfied approximately under certain conditions. In Fig. 2 the function $\Gamma = \gamma^2 \cot^2 \delta$ has been plotted for various values of β and for δ ranging from zero to 90°. The range $90° < \delta < 180°$ is just the mirror image of Fig. 2 on the line $\delta = 90°$. The value $\Gamma = 10$ was rather arbitrarily selected to represent a lower bound for Γ. Under these circumstances, for every value of β there exists a range of directions of motion for which the scattered fields are approximately time-harmonic. Conversely, for each direction of motion, there exists a lower limit on the velocity required to produce time-harmonic fields.

Substituting for k', ρ', ω_0', t' in (4), we may separate the phase into a space part involving x and a time part involving t. From the latter, the Doppler frequency is found to be

$$\omega = \omega_0 \frac{1 + \beta \cos (\alpha - \delta)}{1 - \beta}. \tag{13}$$

Since (13) is subject to the validity of (11), let us take $\gamma^2 \cot^2 \delta = 10$ to ensure satisfaction of (11). Using this relation to eliminate β, the Doppler shift ω/ω_0 is given by

$$\frac{\omega}{\omega_0} = \frac{1 + \sqrt{1 - \dfrac{\cot^2 \delta}{10}} \cos (\alpha - \delta)}{1 - \sqrt{1 - \dfrac{\cot^2 \delta}{10}}}. \tag{14}$$

Plots of ω/ω_0 for two incidence angles are given in Fig. 3. The case $\alpha = 90°$ is not much different from that for $\alpha = 45°$, whereas for $\alpha = 135°$, the branches of the $\alpha = 45°$ plot are interchanged.

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[5] For small β (nonrelativistic velocities), (10) is effectively independent of t, as it should be. For higher β, a first-order correction in $v_0 t$ must be included since we are dealing with phase.

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Bulk Effect Germanium Microwave Modulation

***Abstract*—A bulk effect hot carrier microwave modulator for *Q*-band is described in which the required applied voltages are reduced by an order of magnitude from previous modulators. This is accomplished by the use of semiconductor chips of short lengths.**

The drift velocity of germanium is known to deviate from linearity with respect to applied electric field at high fields and eventually saturate at fields of a few kV/cm.[1] This is due to the charge carriers gaining energy from the applied field and becoming hot, i.e., acquiring an effective temperature higher than that of the crystal lattice. This process depends a great deal on the scattering mechanisms by which the electrons lose momentum and energy to the lattice. At high fields, the saturation of drift velocity has been accounted for by assuming that the electrons which then have high energies can be scattered by the optical phonons. The optical phonons are mainly unexcited at room temperature, so that the electrons lose energy when scattered by them. In the equilibrium situation, it may be shown that the drift velocity of these high-energy electrons is independent of the applied field.[2]

The differential mobilities of the charge carriers in germanium will thus decrease with applied field and eventually be equal to zero when their drift velocities saturate. This property has been utilized in a bulk effect microwave modulator described by Granville.[3] If the energy relaxation time is small compared with the period of the microwaves, the microwave absorption is proportional to the differential conductivity for semiconductors of sufficiently low conductivity. Thus a semiconductor exhibiting this hot carrier effect will become less absorbent to microwaves as the applied electric fields on it are increased and eventually become transparent at high fields. In theory, the speed of such a modulator is limited only by the energy relaxation time of the carriers. The modulator described by Granville and a later one described by Harmatz[4] consisted in each case of a slab of *n*-type germanium placed centrally in a slot in a waveguide across the narrow dimension, parallel to the direction of propagation of the

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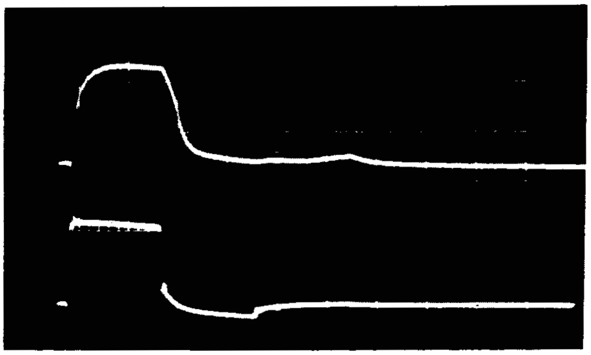

Fig. 1. Upper trace: transmitted microwave pattern; vertical scale: 5 mV per division. Lower trace: applied voltage pulse; vertical scale: 40 V per division. Horizontal scale: 0.5 μs per division. Microwave wavelength: 8.7 mm. Incident CW power: 6 mW.

microwaves. The voltages needed for modulation were very high (about 4 kV for complete transparency), as their slabs were about 1 cm long.

To reduce the voltages needed, a modulator for Q-band microwaves using short chips of both n- and p-type germanium of resistivity between 4.6 and 6.2 $\Omega \cdot$cm was constructed in this laboratory. The modulator consisted of a Q-band waveguide section with a ridge 0.1 inch wide in the center, the distance between the ridge and the top of the waveguide being 0.1 cm. The transitions before and after the ridge were accomplished by tapered sections down to the bottom of the waveguide. A chip of Ge sat in a slight depression on the top of the ridge; the chips used were about 0.1 by 0.1 cm in cross section. Contacts were made to the chip by evaporating Sn onto opposite ends. With the chip in the depression, pulses could be applied to the top Sn contact by a spring-loaded screw pressing down on the top of the chip through an insulated hole in the waveguide roof. The pulses were applied via a miniature Conhex connector soldered to the screw contact.

Pulses of 0.5 to 2 μs duration were applied to the chip at a rate of 50 per second from a Spencer-Kennedy type 503A fast-rise pulse generator, with which pulse amplitudes of up to 150 volts were available. Two n-type Ge chips of lengths 0.175 and 0.122 cm and one p-type Ge chip of length 0.115 cm were used. The VSWR's due to the reflection losses from their front surfaces were 1.33, 1.38, and 1.15, and their standing absorptions were 3.8, 5.6, and 7.25 dB, respectively. When pulses were applied to a chip in the modulator, the observed transmitted microwave power through the chip increased during the pulses. This increase became larger as the applied voltage was increased. The transmitted microwave pattern for the shorter n-type Ge chip is shown together with the applied pulse in Fig. 1. It was found that the transmission pattern varied somewhat with the setting of an E-H tuner before the modulator. This is thought to be due to phase changes of the transmitted microwaves arising from the change in carrier concentration as injected carriers from the contacts pass through the chip in the waveguide. The risetime of the modulator with each chip was of the order of hundreds of nanoseconds, and at best about 100 ns with the shorter n-type Ge chip as shown in Fig. 1.

It was found that up to the highest fields applied, which were about 1 kV/cm, the increase in microwave transmission increased monotonically with the applied field. The increase was fairly linear for the longer n-type and the p-type Ge chips but not for the shorter n-type Ge chip, the slope of the microwave transmission increase against applied voltage curve being greater at lower voltages. The values of microwave conductivity at 1 kV/cm were calculated from the graphs and normalized with respect to the zero field value. The values of normalized microwave conductivity were 0.75 and 0.675 for the long and short n-type Ge chips, and 0.80 for the p-type Ge chip, respectively. These values are comparable to previously obtained values of normalized microwave conductivity at 1 kV/cm.[5]

Thus the possibility of decreasing the required voltages for hot carrier modulators by decreasing the semiconductor length has been demonstrated. In the case of the short chips, the required voltages were decreased by an order of magnitude less than for previous modulators. It is believed that by careful engineering, the chip length could be decreased by an order of magnitude further, so that about 40 to 50 volts would be needed for complete transparency. This would make hot carrier modulators more practical.

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Temperature Dependence of Resonant Frequencies of LiNbO$_3$ Plate Resonators

Abstract—**The temperature coefficient of the resonant frequency and the coupling constant of the LiNbO$_3$ single crystal were found for various crystallographic orientations from plate-shaped resonators in the thickness vibrational mode. The temperature coefficient varied between 7.1 and 9.3 $\times 10^{-5}$ °C^{-1} in any crystal orientation.**

In recent years extensive studies on the piezoelectric properties of LiNbO$_3$ single crystals have been made by several authors. Yamada *et al.* reported the experiment on bar-shaped samples in length extensional modes of vibration,[1] while Warner *et al.* investigated the properties for plate-shaped samples in thickness extensional and thickness shear modes.[2] The temperature dependence of the parameters, however, has been reported only for bar-shaped samples in length modes in the resonant frequency range of a few hundred kHz. In the present experiment, the electromechanical coupling constant and the variation of the resonant frequency against the temperature were measured as functions of the orientation of the crystal for the plate-shaped LiNbO$_3$ resonator in the thickness vibrational mode.

Plate-shaped samples of various crystallographic orientations were cut from the single crystal grown by the Chockralski method. The samples were nearly square, with a side of about 0.5 cm and a thickness of 50 to 250 microns. Gold electrodes were evaporated on the major surfaces of the plate, so the sample was to be excited by the perpendicular field.

The effective electromechanical coupling constant k is defined, according to Warner *et al.*,[2] as

$$\begin{aligned} k &= [X/\tan X]^{1/2} \\ X &= \pi/2 \cdot f_r/f_{ar} \end{aligned} \tag{1}$$

where f_r is the resonant frequency and f_{ar} the antiresonant frequency. The effective electromechanical coupling constant was obtained by measuring the resonant frequency and its higher harmonics.[3] The value thus found is plotted in Fig. 1. The curves in Fig. 1(c) are in good agreement with the data reported by Warner *et al.*[2] except at the vicinity of ±20° from the Z-cut. These disagreements would be due to the difficulty of the observation of weak resonances.

The temperature coefficient of the resonant frequency was measured with the sample put in a Gebrüder Haarke Ultrathermostat for a temperature range of 5° to 95°C. The variation of the resonant frequency with the

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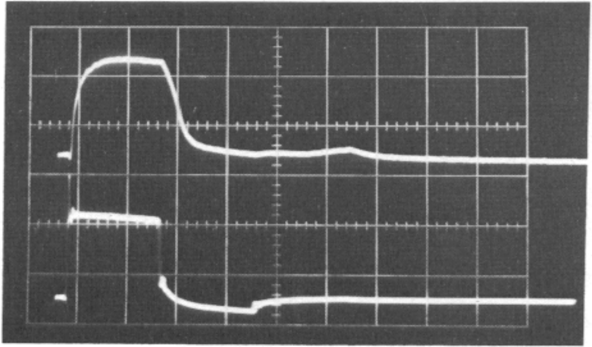

Fig. 1. Upper trace: transmitted microwave pattern; vertical scale: 5 mV per division. Lower trace: applied voltage pulse; vertical scale: 40 V per division. Horizontal scale: 0.5 μs per division. Microwave wavelength: 8.7 mm. Incident CW power: 6 mW.