THE SCIENCE OF MUSIC (HSI2013)
Mid-Term Class Test, Semester 1, 2026/27

This is an open book test. The test is one hour long.

Give your answers to ALL 25 questions on the computer-readable sheet provided, using a soft (2B) pencil to shade the appropriate choice for each question.

1. Which of the following is the least appropriate example of a scientific activity?

   (a) A clarinet player investigating how the material of the clarinet mouthpiece affects the tone of the clarinet.

   (b) A cymbal player playing cymbals of different metals to find out how each material affects the brightness of the sound of the cymbals.

   (c) A physicist rehearsing on a French Horn made of a new metallic alloy.

   (d) A percussionist playing on several different side drums to investigate how the timbre of the drum varies with the diameter of each drum.

   Answer: (c) The clarinet player, cymbal player and percussionist are all performing essentially scientific activities. The physicist is the only one performing an essentially musical activity.

2. Which of the following is the most appropriate example of a technological activity?

   (a) A trombone player playing on a new type of trombone which can also produce sounds which resemble that of a trumpet.

   (b) An oboe player inventing a new type of oboe mouthpiece which can change the tone colour of an oboe to resemble a cor anglais.

   (c) A pianist performing on a new type of portable electronic piano whose tone can be electronically altered to also produce the sound of a harpsichord.

   (d) An engineer rehearsing on a new type of double bass whose frequencies can be increased to also produce 'cello sounds.

   Answer: (b) The trombone player, pianist and engineer are all performing essentially musical activities. The oboe player is the only one performing an essentially technological activity.

3. Which of the following is the least appropriate example of an object undergoing a vibration?

   (a) A flagpole swaying from left to right repeatedly during a strong wind.

   (b) A conductor's arm moving from left to right repeatedly during a concert.

   (c) A scoop of ice cream sliding gradually to a stop on the floor.

   (d) A little boy's head nodding repeatedly during a boring lesson in kindergarten.

   Answer: (c) The flagpole, conductor's arm and little boy's head are each undergoing a repeated motion and hence undergoing a vibration. Only the scoop of ice cream is undergoing a single movement and is therefore not undergoing a vibration.

4. Two trees next to each other are swaying repeatedly from left to right during a storm. The shorter tree undergoes 5 complete cycles in the same time period during which the taller tree undergoes 3 complete cycles. If the taller tree undergoes 6 complete cycles in 7 seconds, calculate the time period during which the shorter tree would undergo 9 complete cycles.

   (a) $\frac{63}{5}$ seconds.

(b) $\frac{7}{2}$ seconds.

(c) $\frac{7}{10}$ seconds.

(d) None of the above.

Answer: (d) Since the taller tree undergoes 6 cycles in 7 seconds, the time it takes to undergo one cycle is given by $\frac{7}{6}$ seconds. The taller tree thus undergoes 3 cycles in $\frac{7}{6}$ seconds times 3 cycles i.e. $\frac{7}{2}$ seconds. The shorter tree undergoes 5 cycles in the same time period, so the duration of each of the shorter tree's cycles is given by $\frac{7}{2}$ seconds divided by 5 i.e. $\frac{7}{10}$ seconds. Therefore the shorter tree would undergo 9 cycles in a duration given by $\frac{7}{10}$ seconds times 9 i.e. $\frac{63}{10}$ seconds.

5. An architect designs a house, and from the architect's plans, a contractor builds the actual house. A businessman views the house and buys it for his family to stay in. A composer composes a symphony, and from the musical score of the symphony an orchestra performs the symphony for an enthusiastic audience during a symphony concert at the Victoria Concert Hall. Which of the following has the same relationship to the composer as the family has to the architect?

   (a) The audience.

   (b) The orchestra.

   (c) The musical score.

   (d) The Victoria Concert Hall.

   Answer: (a). The contractor follows the plans of the house made by the architect and builds the house which is then bought by the businessman for his family. The orchestra reads the musical score of the symphony composed by the composer and performs the symphony for the audience. Therefore the audience has the same relationship to the composer as the family has to the architect.

6. A junior college choir is rehearsing in the college's music room, and as the choir starts to sing the National Day song "Count On Me Singapore", a choir member who is late enters the music room. If counterpoint is to be produced by the choir and the choir member who is late, which of the following songs should the choir member not sing as she enters the music room?

   (a) "We Are Singapore".

   (b) "Where I Belong".

   (c) "Count On Me Singapore".

   (d) "We Will Get There".

   Answer: (c) Since counterpoint is produced when two or more different melodies are sung simultaneously, the choir member who is late should not sing the same song which is being sung by the rest of the choir. Therefore the choir member should not sing "Count On Me Singapore".

7. You are attending a campus concert in which the first item is a violin solo. The sound of this violin solo registers a reading of 69 dB on a sound level meter which you are carrying. The second item in the concert is a rock band whose sound power is 1,000 times more powerful than the sound from the violin solo. What would be the reading on your sound level meter due to the sound from the rock band? (Assume that the sound level meter readings are due only to the performers in the concert.)

(a) 109 dB.

(b) 99 dB.

(c) 89 dB.

(d) None of the above.

Answer: (b) Since a 10 times increase in the sound power would result in an increase of 10 dB in the reading of the sound level meter, an increase of 1,000 times in the sound power i.e. an increase of 10 times 10 times 10 times would result in an increase of 10 dB plus 10 dB plus 10 dB i.e. 30 dB in the sound level meter reading. Therefore the sound level meter reading due to the rock band would be equal to 69 dB plus 30 dB i.e. 99 dB.

8. A flute player plays a note during an orchestral concert with a frequency of 1,760 Hz. A trombone player then plays a note with a frequency of 82.5 Hz. Which of the following best describes the interval between the note played by the trombone player and the note played by the flute player?

(a) Greater than 4 complete octaves but less than 5 complete octaves.

(b) Greater than 3 complete octaves but less than 4 complete octaves.

(c) Greater than 2 complete octaves but less than 3 complete octaves

(d) None of the above.

Answer: (a) An octave is an interval with a ratio of $\frac{2}{1}$ i.e. 2, so if we go up from a musical note by the interval of an octave, the frequency of the musical note should be multiplied by 2. Starting from the trombone player's note of 82.5 Hz, successive multiplications by 2 five times would give us the following frequencies: 82.5 Hz, 165 Hz, 330 Hz, 660 Hz, 1,320 Hz and 2,640 Hz. The interval from 82.5 Hz to 1,760 Hz is therefore greater than 4 complete octaves but less than 5 complete octaves.

9. A bass guitar has its A string tuned to a frequency of 56 Hz, and all its strings are tuned in Equal-tempered semitones as is usual for a bass guitar. A certain note on this bass guitar has a frequency of approximately 266.4 Hz. Which of the following is most likely to be the note with this frequency? (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

(a) Asharp3.

(b) B3

(c) C4.

(d) Csharp4.

Answer: (c) The frequency of the A string of the bass guitar which is the note A1 is 56 Hz, so its A3 note has a frequency given by 56 Hz multiplied by 2 twice i.e. 224 Hz. If we go up from 224 Hz by three Equal-tempered semitones, we should multiply 224 Hz by the ratio of an Equal-tempered semitone i.e. 1.05946 three times. This gives us a frequency of approximately 266.380 Hz, so the note with a frequency of approximately 266.4 Hz is most likely to be the note which is three semitones above A3 i.e. C4.

10. A melody for solo bassoon has a musical score which begins with a time signature of 26/8. One particular bar of this melody begins with 6 semiquavers and ends with 6 quavers and a dotted crotchet. Which of the following combinations of notes would fit exactly into the middle of this bar in agreement with the time signature?

(a) 2 crotchets, a dotted minim and 2 dotted quavers.

(b) 4 dotted quavers, 2 crotchets and 7 semiquavers.

(c) A minim, 3 crotchets and 4 quavers.

(d) A dotted crotchet, a dotted minim and 8 semiquavers.

Answer: (c) Since the time signature is 26/8, this means that each bar of the melody should have the duration equivalent of 26 quavers or 52 semiquavers. The beginning of the bar already has 6 semiquavers and the end of the bar has 6 quavers equivalent to 12 semiquavers and a dotted crotchet equivalent to 6 semiquavers. Therefore the bar already has the duration equivalent of 24 semiquavers, so the middle of the bar should be filled with the duration equivalent of 28 semiquavers. A minim is equivalent to 8 semiquavers and with 3 crotchets (12 semiquavers) and 4 quavers (8 semiquavers) make up a total of 28 semiquavers.

11. You start from the note C5 on a piano keyboard and go downwards by an interval of two and one-quarter octaves to arrive at a second note. You then go upwards from this second note by one and five-sixth octaves to arrive at a third note. Finally you go downwards from this third note by one and two-thirds octaves to arrive at the fourth and final note. Which of the following notes is the correct letter name of the fourth note?

    (a) B2.

    (b) B3.

    (c) Asharp3.

    (d) None of the above.

    Answer: (a) Since an octave consists of 12 semitones, if you go downwards from the starting note C5 by two and one-quarter octaves this means going down by 27 semitones. Therefore the second note is A2. Going up from A2 by one and five-sixth octaves or 22 semitones, the third note is G4. Finally, going down from G4 by one and two-thirds octaves or 20 semitones, you reach the fourth note which is B2.

12. A piece for solo 'cello consists of a musical staff starting with a bass clef. A particular bar of this score has its first note written on the lowest space of the four spaces of the staff, its second note written on the second lowest line of the five lines of the staff, its third note written on the highest space of the four spaces of the staff, and its fourth note written on the middle line of the five lines of the staff. Of the following, which one gives the names of these four notes in the correct sequence from the first note to the fourth note?

    (a) A2, C3, G3 and D3.

    (b) A2, B2, G3 and E3.

    (c) A2, B2, A3 and D3.

    (d) A2, B2, G3 and D3.

    Answer: (d) Since the bass clef shows that the second highest line of the staff is the note F3, the note on the lowest space of the four spaces of the staff is the note A2. The note on the second lowest line of the five lines is the note B2, the note on the highest space of the four spaces is the note G3 and the note on the middle line of the five lines is the note D3.

13. The A4 string of an upright piano is tuned to a frequency of 439 Hz and all its other strings are tuned relative to each other as is normal for a piano. The A string of a viola is tuned to a frequency of 440 Hz and all its other strings are tuned relative to each other in Just fifths as is usual for a viola. A pianist plays a note seven semitones above the middle C of this piano

and a viola player plays the 2nd harmonic of the open C string on this viola. Which of the following is closest to the ratio of the interval between these two notes? Open string means that the notes are not played with a finger on the viola's fingerboard, i.e. they are played with the full length of the respective string vibrating. (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

(a) 1.416.

(b) 1.500.

(c) 1.589.

(d) 2.832.

Answer: (b) Since the note seven semitones above C4 on the piano is the note G4, it is two semitones below its A4 note and its frequency is therefore given by 439 Hz divided by 1.05946 two times. This gives a frequency of approximately 391.1068 Hz. The open C string of the viola is three Just fifths below its A string, so the 2nd harmonic of this string is given by 440 Hz divided by $\frac{27}{8}$ and multiplied by 2, i.e. 260.7407 Hz. The interval between the two notes is thus 391.1068 Hz divided by 260.7407 Hz i.e. approximately 1.500.

14. A 'cello has its A string tuned to a frequency of 220 Hz and all its other strings are tuned relative to each other in Just fifths as is usual for a 'cello. The A string of a bass guitar is tuned to a frequency two octaves below the frequency of the 'cello's A string, and all the bass guitar's strings are tuned in Equal-tempered semitones as is usual for a bass guitar. Which of the following is closest to the ratio of the interval between the note from the open D string of the 'cello and the Fsharp2 note of the bass guitar? Open string means that the notes are not played with a finger on the 'cello's fingerboard, i.e. they are played with the full length of the respective string vibrating. (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

(a) 1.680.

(b) 1.586.

(c) 1.497.

(d) 1.413.

Answer (b) The frequency of the open D string (D3) of the 'cello is one fifth below its A3 string. Therefore it is 220 Hz divided by 3/2 or approximately 146.6667 Hz. The bass guitar's A string (A1) is two octaves below 220 Hz i.e. 55 Hz. Its A2 note is therefore one octave above 55 Hz i.e. 110 Hz. Its Fsharp2 note is three Equal-tempered semitones below its A2 note or 110 Hz divided by 1.05946 three times i.e. approximately 92.4994 Hz. The interval between the note from the open D string of the 'cello and the Fsharp2 note of the bass guitar is therefore 146.6667 Hz divided by 92.4994 Hz i.e. approximately 1.586.

15. Starting from a first note, you go down by an interval of two octaves and a Just fourth to arrive at a second note. Going up from this second note by the interval of a Pythagorean third you arrive at a third note. You then go down from this third note by an octave and a Just fifth to arrive at a fourth note. Finally you go down by a Pythagorean sixth to arrive at a fifth note. Calculate the ratio of the interval between the first note and the fifth note.

(a) $\frac{64}{3}$.

(b) $\frac{32}{3}$.

(c) $\frac{3}{32}$.

(d) None of the above.

Answer: (a) The ratio of a Just fourth is $\frac{4}{3}$, so going down by two octaves and a Just fourth means dividing by 2 twice and then multiplying by $\frac{3}{4}$. The ratio of a Pythagorean third is $\frac{81}{64}$, so going up by a Pythagorean third means multiplying by $\frac{81}{64}$. The ratio of a Just fifth is $\frac{3}{2}$, so going down by an octave and a Just fifth means dividing by 2 and multiplying by $\frac{2}{3}$. The ratio of a Pythagorean sixth is $\frac{27}{16}$ so going down by a Pythagorean sixth means multiplying by $\frac{16}{27}$. The ratio of the interval between the first note and the fifth note is thus given by $\frac{1}{4}$ times $\frac{3}{4}$ times $\frac{81}{64}$ times $\frac{1}{2}$ times $\frac{2}{3}$ times$\frac{16}{27}$. This gives a ratio of $\frac{3}{64}$. Since the ratio should be that of the higher note divided by the lower note, the ratio is $\frac{64}{3}$.

16. When you put your finger at a distance of 17 cm from the nearer end of a vibrating string, it vibrates at a frequency of 425 Hz. Calculate the frequency of the 13th harmonic of the string if the length of the string is 136 cm.

(a) 584.375 Hz.

(b) 637.5 Hz.

(c) 731.25 Hz.

(d) None of the above.

Answer: (d) 17 cm is one-eighth of 136 cm, so the string must be vibrating at its 8th harmonic, and hence the fundamental frequency of the string is equal to 425 Hz divided by 8 i.e. 53.125 Hz. The thirteenth harmonic of 53.125 Hz is equal to 53.125 Hz times 13 i.e. 690.625 Hz.

17. A 40 cm long string labelled P is vibrating at a frequency of 2,160 Hz with x nodes between its two ends (not counting the nodes at both ends). A second string labelled Q which is 64 cm long is vibrating at a frequency of 1,200 Hz with 8 antinodes between its two ends. String P is then lengthened by 20% and vibrates at 1,000 Hz with y nodes between its two ends (not counting the nodes at both ends). Find the ratio *x:y.* (Assume that strings P and Q are identical in all respects except length.)

(a) 9:5.

(b) 5:3.

(c) 3:2.

(d) None of the above.

Answer: (d) Since the string Q has 8 antinodes it is at its 8th harmonic. Its fundamental frequency is thus equal to 1,200 Hz divided by 8 i.e. 150 Hz. The fundamental frequency of string P is thus equal to 150 Hz times $\frac{64}{40}$ i.e. 240 Hz. 2,160 Hz divided by 240 Hz is equal to 9, so string P is vibrating at its 9th harmonic and has 9 antinodes or 8 nodes between its two ends. When string P is lengthened by 20%, its fundamental frequency will be equal to 240 Hz times $\frac{100}{120}$ i.e. 200 Hz. Since 1,000 Hz divided by 200 Hz is equal to 5, the lengthened string P is vibrating at its 5th harmonic and has 5 antinodes or 4 nodes between its two ends. The ratio *x:y* is therefore equal to 8:4 or 2:1.

18. The distance between a pair of adjacent nodes of a 144-cm long vibrating string is 36 cm. What is the wavelength of the sound wave produced by the vibrating string travelling towards an audience if the string has a fundamental frequency of 82.5 Hz? (Assume that the velocity of sound in air is 330 m/s.)

(a) 160 cm.

(b) 100 cm.

(c) 50 cm

(d) None of the above.

Answer: (b) The length of each vibrating segment of the string is 36 cm, so the number of such segments in the string is given by 144 cm divided by 36 cm i.e. 4. Therefore the string has 4 antinodes and is vibrating at its 4th harmonic, so its frequency is given by 82.5 Hz multiplied by 4 i.e 330 Hz. Hence the wavelength of the sound wave is given by 330 m/s divided by 330 Hz i.e. 1.0 m or 100 cm.

19. A 'cellist plays a musical note on his 'cello which produces a sound wave travelling towards a listener with a wavelength of exactly 60 cm. Which of the following musical notes is most likely to be the note played by the 'cellist? (Assume that the velocity of sound in air is 330 m/s. Assume also that the notes played by the 'cello are part of the Just scale and that the A string on the 'cello is tuned to a frequency of 220 Hz.)

    (a) Csharp5.

    (b) C5.

    (c) B4.

    (d) None of the above.

    Answer: (a) Since the 'cello's musical note has a wavelength of 60 cm or 0.6 m in air, its frequency is equal to the speed of sound in air i.e. 330 m/s divided by 0.6 m i.e 550 Hz. The 'cello's A4 note is one octave above its A3 note, and is therefore equal to 220 Hz multiplied by 2 i.e. 440 Hz. Going up from the A4 note by a Just third, we multiply 440 Hz by the ratio $\frac{5}{4}$ to obtain the frequency 550 Hz. The note with a frequency of 550 Hz which is a Just third above A4 must be Csharp5, so the note played is the note Csharp5.

20. The succession of intervals for a certain Balinese pentatonic scale is as follows: semitone, tone, 2 tones, semitone, 2 tones, which makes up a total of 12 semitones. Starting this Balinese pentatonic scale from the note E4, which of the following is the sequence of notes based on this Balinese pentatonic scale?

    (a) E4, F4, G4, B4, D5 and E5.

    (b) E4, F4, G4, Gsharp4, C5 and E5.

    (c) E4, F4, G4, B4, C5 and E5.

    (d) E4, Fsharp4, G4, B4, C5 and E5.

    Answer: (c) If we start from the note E4, the correct succession of intervals i.e. semitone, tone, 2 tones, semitone and 2 tones, will produce the following sequence of notes: E4, F4, G4, B4, C5 and E5.

21. An ancient musical instrument plays a musical note which has a spectrum showing that all its harmonics, odd and even, are present up to its 22nd harmonic. The spectrum of a square wave shows its fundamental frequency and all its harmonics up to its 21st harmonic. The frequency of the 8th line from the left in the spectrum of the musical instrument's note is the same as that of the 6th line from the left in the spectrum of the square wave. If the fundamental frequency of the square wave is 96 Hz, calculate the frequency of the 12th line from the left in the spectrum of the musical instrument's note. (Assume that the frequencies in each spectrum increase from left to right.)

    (a) 864 Hz.

(b) 1,296 Hz.

(c) 1,872 Hz.

(d) None of the above.

Answer: (d) The 6th line from the left in the spectrum of the square wave is its 11th harmonic, so its frequency is equal to 96 Hz times 11 i.e. 1,056 Hz. The 8th line from the left in the spectrum of the musical instrument's note is its 8th harmonic, so its fundamental frequency is given by 1,056 Hz divided by 8 i.e. 132 Hz. The 12th line from the left in the spectrum of the musical instrument's note is its 12th harmonic, which is equal to 132 Hz times 12 i.e. 1,584 Hz.

22. An open pipe labelled E is sliced into two parts. One part is an open pipe two-thirds the length of pipe E labelled F1. The other part which is one-third the length of pipe E has one end closed up to make a closed pipe labelled F2. Which of the following statements concerning pipes E, F1 and F2 are correct?
(i) Both pipe F1 and pipe F2 have the same fundamental frequencies.
(ii) The frequency of pipe E vibrating with 2 antinodes (not counting the antinodes at both ends) between its two ends is the same as that of pipe F1 vibrating with 2 nodes between its two ends.
(iii) Pipe E has to be shortened to one-ninth its original length in order for its fundamental frequency to be twice the frequency produced by pipe F2 vibrating with 1 node (not counting the node at one end) between its two ends.
(iv) The ratio of the wavelength of the sound wave produced by pipe F1 vibrating with 5 nodes between its two ends to that produced by pipe F2 vibrating with 3 antinodes (not counting the antinode at one end) between its two ends is 5:7.

(a) All statements are correct.

(b) Statements (i),(ii) and (iii) are correct.

(c) Statements (i), (iii) and (iv) are correct.

(d) Only statements (i) and (ii) are correct.

Answer: (b) Let the fundamental frequency of pipe E be f Hz. Pipe F1 which is an open pipe two-thirds the length of pipe E will have a fundamental frequency of $\frac{3f}{2}$ Hz. An open pipe which is one-third the length of pipe E will have a fundamental frequency of $3f$ Hz so the closed pipe F2 will have a fundamental frequency of $\frac{3f}{2}$ Hz. Both pipe F1 and pipe F2 have fundamental frequency of $\frac{3f}{2}$ Hz, so statement (i) is correct. When pipe E vibrates with 2 antinodes or 3 nodes, it is at its third harmonic with frequency $3f$ Hz. When pipe F1 vibrates with 2 nodes, it is at its second harmonic with frequency $3f$ Hz as well, so statement (ii) is correct. When pipe E is shortened to one-ninth its original length, its fundamental frequency is $9f$ Hz. When pipe F2 vibrates with 1 node, it is at its third harmonic with frequency $\frac{9f}{2}$ Hz. So statement (iii) is correct. When F1 vibrates with 5 nodes, it is at its fifth harmonic with frequency $\frac{15f}{2}$ Hz. When F2 vibrates with 3 antinodes, it is at its seventh harmonic with frequency of $\frac{21f}{2}$ Hz. The ratio of the frequency of pipe F1 vibrating with 5 nodes to that of pipe F2 vibrating with 3 antinodes is thus 15:21 or 5:7. As frequency is inversely proportional to wavelength, the ratio of the corresponding wavelengths is thus 7:5, so statement (iv) is incorrect.

23. A string and a closed pipe have the same fundamental frequency. The string is then lengthened to 125% of its original length. When the lengthened string vibrates with 5 nodes (not counting the nodes at both ends) the wavelength of the sound wave it produces is exactly 0.11 metres.

Calculate the frequency produced by the closed pipe when it vibrates with 6 nodes (not counting the node at one end). (Assume that the velocity of sound in air is 330 m/s.)

(a) 3,750 Hz.

(b) 7,800 Hz.

(c) 8,125 Hz

(d) None of the above.

Answer: (c) The frequency of the sound wave produced by the lenghtened string is given by 330 m/s divided by 0.11 m i.e. 3,000 Hz. Therefore since it is vibrating with 5 nodes it has 6 antinodes and is at its sixth harmonic so its fundamental frequency is equal to 3,000 Hz divided by 6 i.e. 500 Hz. The fundamental frequency of the original string is thus equal to 500 Hz multiplied by $\frac{125}{100}$ i.e. 625 Hz. The closed pipe is vibrating with 6 nodes so it will be at its 13th harmonic which is equal to 625 Hz times 13 i.e. 8,125 Hz.

24. An open pipe labelled A is cut into 8 short open pipes. 5 of these short open pipes are joined up and one end closed up to make a closed pipe labelled B. If the fundamental frequency of A is 103.125 Hz, calculate the wavelength of the sound wave produced by B when it vibrates with 2 nodes (not counting the node at one end) between its two ends. (Assume that the velocity of sound in air is 330 m/s.)

(a) 40 cm.

(b) 80 cm.

(c) 200 cm.

(d) None of the above.

Answer: (b) An open pipe the same length as B would have a fundamental frequency equal to 103.125 times $\frac{8}{5}$ Hz i.e. 165 Hz. Therefore the closed pipe B has a fundamental frequency given by 165 Hz divided by 2 i.e. 82.5 Hz. When B vibrates with 2 nodes, it is at its 5th harmonic and its frequency is thus given by 82.5 Hz times 5 i.e. 412.5 Hz. The resulting sound wave will have a wavelength equal to 330 m/s divided by 412.5 Hz i.e. 0.8 m or 80 cm.

25. Arrange the following harmonics in order of increasing frequency:
(i) The eighth harmonic frequency of an open pipe W of length d cm.
(ii) The sixth harmonic frequency of an open pipe X of length $\frac{5d}{6}$ cm.
(iii) The ninth harmonic frequency of a closed pipe Y of length $\frac{3d}{5}$ cm.
(iv) The fifth harmonic frequency of a closed pipe Z of length $\frac{2d}{7}$ cm.

(a) (i), (ii), (iii) and (iv).

(b) (ii), (iii), (iv) and (i).

(c) (ii), (iv), (i) and (iii).

(d) None of the above.

Answer: (d) Let f Hz be the fundamental frequency of the open pipe W of length d cm. The eighth harmonic frequency of W is thus equal to $8f$ Hz. The open pipe X of length $\frac{5d}{6}$ cm has a fundamental frequency equal to f Hz times $\frac{6d}{5d}$ i.e. $\frac{6f}{5}$ Hz, so the sixth harmonic frequency of X is equal to $\frac{6f}{5}$ Hz times 6 i.e. $\frac{36f}{5}$ Hz. An open pipe with the length $\frac{3d}{5}$ cm will have a fundamental frequency given by f Hz times $\frac{5d}{3d}$ i.e. $\frac{5f}{3}$ Hz, and therefore the closed pipe Y which has the same length of $\frac{3d}{5}$ cm will have a fundamental frequency half of $\frac{5f}{3}$ Hz i.e. $\frac{5f}{6}$ Hz. The ninth harmonic frequency of Y is thus equal to $\frac{5f}{6}$ Hz times 9 i.e. $\frac{45f}{6}$ Hz. The closed

pipe Z of length $\frac{2d}{7}$ cm has a fundamental frequency given by $\frac{5f}{6}$ Hz times $\frac{3d}{5}$ divided by $\frac{2d}{7}$ i.e. $\frac{7f}{4}$ Hz. Therefore the frequency of the fifth harmonic of Z is thus equal to $\frac{7f}{4}$ Hz times 5 i.e. $\frac{35f}{4}$ Hz. Hence the harmonics in order of increasing frequency are (ii), (iii), (i) and (iv).

END OF TEST PAPER