Answers to Tutorial No 2.5, Semester 1, 2010/11

1. A boy is standing some distance away from a high wall, and between him and the wall is a tall tree. He sees a lightning bolt striking the tree, and 0.35 seconds later, he hears the sound of thunder which is caused by this lightning bolt. 0.95 seconds after he sees the lightning bolt, he hears the sound of thunder again, which is due to the reflection of the thunder from the high wall. Calculate the distance of the tree and the wall from the boy. A bird which is sitting on a branch in the tree sings a note which has a wavelength of 30 cm. What is the frequency of the note sung by the bird? (Assume that the speed of sound in air is 330 metres per second and that that light from the lightning flash is received instantaneously.)
   Answer: The direct sound of the thunder was heard 0.35 seconds after the flash of the lightning bolt was seen. Therefore if the speed of sound in air is 330 metres per second, the distance from the tree to the boy is given by 330 metres per second times 0.35 seconds i.e. 115.5 metres. If we denote the distance between the tree and the high wall as d metres, then the sound of the reflection of the thunder travels a distance d metres to the wall, then the same distance d metres back to the tree, and finally 115.5 metres to the boy, giving a total distance of $2d$ plus 115.5 metres. But this distance must also be equal to 330 metres per second times 0.95 seconds which is equal to 313.5 metres, so we have $2d$ metres equal to 313.5 metres minus 115.5 metres, giving a value of 99 metres for the distance d metres between the tree and the high wall. The distance from the wall to the boy is thus equal to 115.5 metres plus 99 metres i.e. 214.5 metres. If the wavelength of the note from the bird is 30 cm or 0.3 metres, then the frequency of the note is equal to 330 metres per second divided by 0.3 metres i.e. 1,100 Hz.

2. A closed pipe is vibrating with 3 nodes between its two ends (not counting the node at one end) at a frequency of 4,200 Hz. When the closed pipe is vibrating with 2 nodes between its two ends, its frequency of vibration is the same as that of an open pipe vibrating with 6 nodes between its two ends. When this open pipe vibrates with 3 nodes between its two ends, it has the same frequency as a string vibrating with 8 antinodes between its two ends. What are the fundamental frequencies of the closed pipe, the open pipe, and the string?
   Answer: When the closed pipe has 3 nodes, it is at its 7th harmonic, and hence its fundamental frequency is equal to 4,200 Hz divided by 7 i.e. 600 Hz. When the closed pipe is vibrating with two nodes, it is at its 5th harmonic, and hence its frequency will be equal to 600 Hz times 5 i.e. 3,000 Hz. This is the same frequency as the open pipe vibrating

with 6 nodes and which is thus at its 6th harmonic. The fundamental frequency of the open pipe is thus equal to 3,000 Hz divided by 6 i.e. 500 Hz. When the open pipe has 3 nodes, it is at its 3rd harmonic, which has a frequency equal to 500 Hz times 3 i.e. 1,500 Hz. This is the same frequency as the string with 8 antinodes and which is at its 8th harmonic. The fundamental frequency of the string is therefore equal to 1,500 Hz divided by 8 i.e. 187.5 Hz.

3. An open pipe has a fundamental frequency of 350 Hz. In the spectrum of the note produced by this open pipe, the 5th line from the left in the spectrum has the same frequency as the 4th line from the left in the spectrum of a closed pipe. What is the fundamental frequency of the closed pipe? If the length of the open pipe is p cm, what is the length of the closed pipe? The closed pipe is now cut into two pieces of equal length. What would be the fundamental frequencies of these two shorter pipes?
Answer: In the spectrum of the open pipe, the 5th line from the left is its 5th harmonic, which has a frequency equal to 350 Hz times 5 i.e. 1,750 Hz. The 4th line from the left in the spectrum of the closed pipe is its 7th harmonic, and hence the fundamental frequency of the closed pipe is equal to 1,750 Hz divided by 7 i.e. 250 Hz. The open pipe of length p cm has a fundamental frequency of 350 Hz, and hence a closed pipe of the same length would have a fundamental frequency equal to 350 Hz divided by 2 i.e. 175 Hz. Therefore the closed pipe which has a fundamental frequency of 250 Hz has a length given by p cm times $\frac{175}{250}$ cm i.e. $\frac{7p}{10}$ cm. If the closed pipe is cut into two pieces of equal length, the shorter closed pipe is half the length of the original closed pipe and thus its fundamental frequency is equal to 250 Hz times 2 i.e. 500 Hz. The shorter open pipe has the same length and thus has a frequency double that of the shorter closed pipe or 500 Hz times 2 i.e. 1,000 Hz.