Answers to Tutorial No 2, Semester 2, 2023/24

1. When you place your finger 18 cm from one end of a string which is 54 cm long, it vibrates at a frequency of 360 Hz. Calculate the fundamental frequency of the string. If the string's length is then increased by 25%, how far should your finger be placed from the nearer end of the string so that the string will vibrate with a frequency of 480 Hz?
   Answer: Since 18 cm is one-third of the string's length of 54 cm, the string must be vibrating at its 3rd harmonic frequency, which means that the fundamental frequency of the string is equal to 360 Hz divided by 3 i.e. 120 Hz. When the length of the string is increased by 25%, its length will be given by 54 cm times 1.25 i.e. 67.5 cm. Therefore the fundamental frequency of the string will be given by 120 Hz times $\frac{54}{67.5}$ i.e. 96 Hz. When the string vibrates with a frequency of 480 Hz, it must be vibrating at its 5th harmonic since 480 Hz divided by 96 Hz is 5. Your finger should thus be placed at a distance equal to one-fifth of 67.5 cm i.e. 13.5 cm from the nearer end.

2. A string which is 75 cm long is vibrating with 5 antinodes between its two ends at a frequency of 1,500 Hz. A second string is vibrating at a frequency of 900 Hz with 6 antinodes between its two ends. What is the length of the second string? If a third string of length

90 cm is vibrating at a frequency of 2,250 Hz, calculate the number nodes which this third string has between its two ends (not counting the nodes at both ends). (Assume that the three strings are similar in all respects except for length.)

Answer: Since the first string has 5 antinodes it is at its 5th harmonic, and its fundamental frequency is thus equal to 1,500 Hz divided by 5 i.e. 300 Hz. The second string has 6 antinodes so it must be at its 6th harmonic, and its fundamental frequency is thus given by 900 Hz divided by 6 i.e. 150 Hz. The length of the second string is therefore equal to 75 cm times $\frac{300}{150}$ i.e. 150 cm. The third string is 90 cm long, so its fundamental frequency is given by 300 Hz times $\frac{75}{90}$ i.e. 250 Hz. Since 2,250 Hz divided by 250 Hz is equal to 9, the third string must be vibrating at its 9th harmonic and therefore has 9 antinodes and 8 nodes between its two ends (not counting the nodes at both ends).

3. If we start from a first musical note and then go up by the interval of a Just sixth, we will arrive at a second note. If we start again from the same first note, but now go up again this time by the interval of a Pythagorean sixth, we will arrive at a third note. Which of these two notes i.e. the second and third notes, has the higher frequency, and what is the ratio of the interval between these two notes? If the frequency of the first note is 120 Hz, what are the frequencies of the second and third notes? If we start again from the same first note of frequency 120

Hz, but now go down instead of up by the same two intervals i.e. the Just sixth and the Pythagorean sixth, calculate the frequencies of the second and third notes.

Answer: A Just sixth has the ratio of $\frac{5}{3}$ which is approximately equal to 1.667 and the ratio of a Pythagorean sixth is $\frac{27}{16}$ which is equal to 1.6875. Therefore the third note has a higher frequency than the second note. If the frequency of the first note is 120 Hz, the frequency of the second note is equal to 120 Hz times $\frac{5}{3}$ i.e. 200 Hz, and the frequency of the third note is equal to 120 Hz times $\frac{27}{16}$ i.e. 202.5 Hz. The interval between these two notes has a ratio which is given by $\frac{27}{16}$ divided by $\frac{5}{3}$ which is the same as $\frac{27}{16}$ multiplied by $\frac{3}{5}$ which is equal to $\frac{81}{80}$. if we down instead of up, the frequency of the second note would now be given by 120 Hz divided by $\frac{5}{3}$ which is the same as 120 Hz times $\frac{3}{5}$ i.e. 72 Hz, and the frequency of the third note would now be given by 120 Hz times $\frac{16}{27}$ i.e. approximately 71.111 Hz.

4. The common pentatonic scale often used in the folk songs of many musical cultures can be found on a piano by playing only the black notes on its keyboard in sequence. The term "pentatonic" (meaning "five notes") scale is so-called because the scale consists of only five notes (not counting the note one octave above the beginning of the scale). The common pentatonic scale is constructed using the following sequence of intervals: tone, tone, three semitones, tone, followed by three semitones, arriving at the

final note exactly one octave or 12 semitones above the starting note. A different type of pentatonic scale is the Balinese gamelan pentatonic scale which has a different sequence of intervals: semitone, tone, 2 tones, semitone, 2 tones, making up a total of 12 semitones. If we start from the note A just below Middle C, what are the letter names of the notes making up these two different pentatonic scales? If you start instead from the G just above Middle C, what are the names of the notes making up these two pentatonic scales?

Answer: If we start from the note A, the next note in the common pentatonic scale is a tone above i.e. the note B, and the next note is also a tone above i.e. the note Csharp/Dflat. Going up three semitones brings us to the next note E, and up another tone brings us to the note Fsharp/Gflat, and up another three semitones brings us back to the note A one octave above the starting A. Starting from the first A again, the next note in the Balinese pentatonic scale one semitone up is Asharp/B flat. Going up by a tone gives us C, and 2 tones up gives us E. Another semitone up gives us F, and up by 2 tones arrives at the A one octave above the starting A. Starting from the note G instead of A, the common pentatonic scale gives the notes A, B, D, E and G again. The Balinese pentatonic scale starting from G gives us Gsharp/Aflat, Asharp/Bflat, D, Dsharp/Eflat and G again.

5. All the strings of a 'cello are tuned in Just fifths as

is normal for a 'cello, and its A string is tuned to a frequency of 220 Hz. A bass guitar's four strings are tuned relative to each other as usual for a bass guitar, and its A string is tuned to a frequency of 55 Hz. Calculate the frequencies of the 'cello's C string and its G string and the ratio of the interval between these two frequencies. Calculate also the frequencies on the bass guitar of the two musical notes which are equivalent to these two notes on the violin, and give the ratio between these two notes on the bass guitar. What is the ratio of the interval between the frequencies of the bass guitar's G2 note and the 'cello's G2 note? (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

Answer: The 'cello's C string which is the note C2 is three Just fifths (of which the ratio is $\frac{3}{2}$) below the cello's A string. Therefore the C string's frequency is given by 220 Hz divided by $\frac{3}{2}$ three times, which is the same as multiplying 220 Hz by $\frac{8}{27}$ i.e. approximately 65.185 Hz. Its G string which is the note G2 is two Just fifths below 220 Hz so its frequency is given by multiplying 220 Hz by $\frac{4}{9}$ i.e. approximately 97.778 Hz. The ratio between the frequencies of the 'cello's C and G strings is simply a Just fifth or $\frac{3}{2}$ or 1.5. The semitones on the bass guitar are all Equal-tempered, so they have a ratio of approximately 1.05946. The guitar's A string is the note A1 with frequency 55 Hz, so the guitar's C2 note is three semitones above its A1 note, and its frequency is equal to 55 Hz multiplied by 1.05946 three times, i.e. approximately 65.406 Hz. The guitar's G2 note is 7 semitones above its C2 note, so its frequency is approximately equal to 65.406 Hz multiplied by 1.05946 seven times i.e. approximately 97.996 Hz. Hence the ratio between the guitar's C2 note and its G2 note is given by the ratio of seven Equal-tempered semitones i.e. approximately 1.4983. The guitar's G2 note has a frequency of approximately 97.996 Hz, and the 'cello's G string which is also its G2 note has a frequency of approximately 97.778 Hz. Therefore the ratio between these two notes is approximately given by 97.996 Hz divided by 97.778 Hz i.e. approximately 1.0022.

6. The graph of the spectrum of a musical sound has vertical lines on the x-axis which represent the fundamental frequency and harmonics of the sound. The lengths of the lines represent the amplitudes of the harmonics, and their positions on the horizontal x-axis represent their frequencies. A newly invented musical wind instrument produces a note which has a spectrum showing its fundamental frequency and all its harmonics up to the 19th harmonic, and all harmonics (odd and even) are present in this spectrum. The 5th line from the left in this spectrum has the same frequency as the 8th line from the left in the spectrum of a square wave. If the frequency of the 6th line in the spectrum of the square wave is 1,320 Hz, calculate the frequencies of the 7th and 17th lines from the left in the spectrum of the musical instrument's note.

   Answer: The spectrum of a square wave only contains odd harmonics, so the 6th line from the left in the square wave's spectrum is its 11th harmonic, which has a frequency of 1,320 Hz. The fundamental frequency of the square wave is hence equal to 1,320 Hz divided by 11 i.e. 120 Hz. The 8th line from the left in the square wave's spectrum is its 15th harmonic, so its frequency is equal to 120 Hz times 15 i.e. 1,800 Hz. The 5th line from the left in the musical instrument's note's spectrum is its 5th harmonic, so the fundamental frequency of its note is equal to 1,800 Hz divided by 5 i.e. 360 Hz. Since the 7th and 17th lines in the spectrum of the musical instrument's note are its 7th and 17th harmonics respectively, their frequencies are equal to 360 Hz times 7 i.e. 2,520 Hz and 360 Hz times 17 i.e. 6,120 Hz respectively.

Scientific Inquiry discussion points

1. The Pythagorean scale, said to be first defined by the Greek mathematician after whom it is named, was based on the ratios of just two intervals -the octave (2/1) and the fifth (3/2). Its simplicity of construction served as the basis of the music of civilisations such as ancient Greece and China. The Pythagorean scale's drawback was that the ratio of the third was complex (81/64) and deemed unsatisfactory by many. As the interval of the third became more important, proponents of the Just scale, in which the ratio of the third was 5/4 instead of 81/64, much preferred it to the Pythagorean scale, as ratios with small numbers were considered by the Greeks to be more beautiful than ratios with large

numbers. The proponents of the Pythagorean scale of course disagreed strongly. Here we see the objective scientific inquiry of Pythagoras coming into conflict with subjective aesthetic judgement. Can you think of other examples in which subjective judgements come into conflict with objective scientific inquiry?

There are many examples of objective scientific inquiry coming into conflict with subjective perception. For example, many ancient civilisations believed in a system in which the sun revolved around the earth, as this seems to be supported by our subjective observation of the sun's motion. However, more detailed study of the sun's motion showed this was untenable, and eventually astronomers could explain it's actual motion only by adopting the theory that the earth revolves around the sun. A more recent example is the subjective belief that some races are superior to other races, simply because of factors such as the colour of their skin or other physical features. Modern understanding of genetics has shown that there is no actual scientific objective basis for the notion of racial superiority.