Answers to Tutorial No 2, Semester 1, 2024/25

1. When your finger is placed 15 cm from one end of a string which is 75 cm long, the string vibrates at a frequency of 600 Hz. What is the fundamental frequency of the string? If we increase the string's length by 20%, calculate the distance your finger should be placed from the nearer end of the string to make the string vibrate with a frequency of 800 Hz.
   Answer: 15 cm is one-fifth of the string's length of 75 cm, so the string is vibrating at its 5th harmonic frequency. Hence this means that the fundamental frequency of the string is equal to 600 Hz divided by 5 i.e. 120 Hz. On increasing the length of the string by 20%, its length will be equal to 75 cm times 1.2 i.e. 90 cm. The fundamental frequency of the string will then be given by 120 Hz times $\frac{75}{90}$ i.e. 100 Hz. When the string vibrates with a frequency of 800 Hz, it is vibrating at its 8th harmonic since 800 Hz divided by 100 Hz is 8. Your finger should thus be placed at a distance equal to one-eighth of 90 cm i.e. 11.25 cm from the nearer end.

2. A string 50 cm long is vibrating with 6 antinodes between its two ends at a frequency of 1,440 Hz. If a second string is vibrating at a frequency of 1,600 Hz with 8 antinodes between its two ends, what is the length of the second string? A third string of length 80 cm is vibrating at a frequency of 750 Hz.

What is the number of nodes which this third string has between its two ends (not counting the nodes at both ends)? (Assume that the three strings are similar in all respects except for length.)

Answer: The first string has 6 antinodes so it is at its 6th harmonic, and its fundamental frequency is given by 1,440 Hz divided by 6 i.e. 240 Hz. Since the second string has 8 antinodes, it must be at its 8th harmonic, and hence its fundamental frequency is equal to 1,600 Hz divided by 8 i.e. 200 Hz. Therefore the length of the second string is given by 50 cm times $\frac{240}{200}$ i.e. 60 cm. Since the third string is 80 cm long, its fundamental frequency is given by 240 Hz times $\frac{50}{80}$ i.e. 150 Hz. 750 Hz divided by 150 Hz is equal to 5, so the third string must be vibrating at its 5th harmonic. The third string thus must have has 5 antinodes and 4 nodes between its two ends (not counting the nodes at both ends).

3. Starting from a first musical note and then going up by the interval of a Just seventh, we will arrive at a second note. Starting again from the same first note, and going up again this time by the interval of a Pythagorean seventh, we will arrive at a third note. Of these two notes i.e. the second and third notes, which one has the higher frequency? Calculate the ratio of the interval between these two notes. If the frequency of the first note is 160 Hz, what are the frequencies of the second and third notes? Starting again from the same first note with a frequency of 160 Hz, and going down instead of up by

the same two intervals i.e. the Just seventh and the Pythagorean seventh, what would be the frequencies of the second and third notes?

Answer: Since a Just seventh has the ratio of $\frac{15}{8}$ which is equal to 1.875 and the ratio of a Pythagorean seventh is $\frac{243}{128}$ which is approximately equal to 1.8984, the third note has a higher frequency than the second note. If the frequency of the first note is 160 Hz, the frequency of the second note is given by 160 Hz times $\frac{15}{8}$ i.e. 300 Hz, and the frequency of the third note is given by 160 Hz times $\frac{243}{128}$ i.e. 303.75 Hz. The interval between these two notes has a ratio which is given by $\frac{243}{128}$ divided by $\frac{15}{8}$ which is the same as $\frac{243}{128}$ multiplied by $\frac{8}{15}$ which is equal to $\frac{243}{240}$ which can be reduced to $\frac{81}{80}$. if we go down instead of up, the frequency of the second note would now be given by 160 Hz divided by $\frac{15}{8}$ which is the same as 160 Hz times $\frac{8}{15}$ i.e. approximately 85.333 Hz, and the frequency of the third note would now be given by 160 Hz times $\frac{128}{243}$ i.e. approximately 84.2798 Hz.

4. On a piano keyboard, the common pentatonic scale often used in the folk songs of many musical cultures can be found by playing only the black notes on the keyboard in sequence. The term "pentatonic" which means "five notes" is the name of this scale because it consists of only five notes (not counting the note one octave above the beginning of the scale). The common pentatonic scale has the following sequence of intervals: tone, tone, three semitones, tone, three semitones, arriving at the final note exactly one oc-

tave or 12 semitones above the starting note. Another type of pentatonic scale is the Balinese gamelan pentatonic scale which has a different sequence of intervals: semitone, tone, 2 tones, semitone, 2 tones, making up a total of 12 semitones. If we start from the note E just above Middle C, what are the letter names of the notes making up these two different pentatonic scales? If we start instead from the A just below Middle C, what are the names of the notes making up these two pentatonic scales?

Answer: Starting from the note E, the next note in the common pentatonic scale is a tone above i.e. the note Fsharp/Gflat, and the next note is also a tone above i.e. the note Gsharp/Aflat. Going up three semitones brings us to the next note B, and up another tone brings us to the note Csharp/Dflat, and up another three semitones brings us back to the note E one octave above the starting E. Starting from the first E again, the next note in the Balinese pentatonic scale one semitone up is F, and going up by a tone gives us G, and 2 tones up gives us B. Another semitone up gives us C and up by 2 tones arrives at the E one octave above the starting E. Starting from the note A instead of E, the common pentatonic scale gives the notes A, B, Csharp/Dflat, E, Fsharp/Gflat and A again. The Balinese pentatonic scale starting from A gives us A, Asharp/Bflat, C, E, F and A again.

5. The strings of a viola are tuned in Just fifths as is usual for a viola, and the viola's A string is tuned to a

frequency of 440 Hz. A guitar's six strings are tuned relative to each other as is usual for a guitar, and its A string is tuned to a frequency of 110 Hz. What are the frequencies of the viola's G string and its D string and the ratio of the interval between these two frequencies? What are the frequencies of the two musical notes on the guitar which are equivalent to these two notes on the viola, and what is the ratio between these two notes on the guitar? Calculate the ratio of the interval between the frequencies of the guitar's B3 note and the viola's D4 note. (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

Answer: Since the viola's G string which is the note G3 is two Just fifths (of which the ratio is $\frac{3}{2}$) below the viola's A string, the G string's frequency is given by 440 Hz divided by $\frac{3}{2}$ two times, which is the same as multiplying 440 Hz by $\frac{4}{9}$ i.e. approximately 195.556 Hz. Its D string which is the note D4 is one Just fifth below 440 Hz so its frequency is given by multiplying 440 Hz by $\frac{2}{3}$ i.e. approximately 293.333 Hz. The ratio between the frequencies of the viola's G and D strings is a Just fifth or $\frac{3}{2}$ or 1.5. Since the semitones on the guitar are all Equal-tempered, they have a ratio of approximately 1.05946. The guitar's A string is the note A2 with frequency 110 Hz, and its A3 note's frequency is double this i.e. 220 Hz. The guitar's G3 note is two semitones below its A3 note, and its frequency is thus equal to 220 Hz divided by 1.05946 two times, i.e. approximately 195.999 Hz. The guitar's D4 note is 5 semitones above its A3 note, so its frequency is approximately equal to 220 Hz multiplied by 1.05946 five times i.e. approximately 293.660 Hz. The ratio between the guitar's G3 note and its D4 note is thus given by the ratio of seven Equal-tempered semitones i.e. approximately 1.4983. The guitar's B3 note being two semitones above A3 has a frequency given by 220 Hz times 1.05946 twice i.e. approximately 246.940 Hz, and the viola's D string which is also its D4 note has a frequency of approximately 293.333 Hz. Therefore the ratio between these two notes is approximately given by 293.333 Hz divided by 246.940 Hz i.e. approximately 1.1879.

6. The spectrum of a musical sound is represented by a graph which has vertical lines on the x-axis representing the fundamental frequency and harmonics of the sound. The positions of the lines on the horizontal x-axis represent their frequencies and the lengths of the lines represent the amplitudes of the harmonics. A newly discovered ancient musical wind instrument produces a note which has a spectrum showing its fundamental frequency and all its harmonics up to the 21st harmonic, and all harmonics (odd and even) are present in this spectrum. The 8th line from the left in this spectrum has the same frequency as the 9th line from the left in the spectrum of a square wave. If the frequency of the 5th line in the spectrum of the square wave is 1,440 Hz, what are the frequencies of the 4th and 15th lines from the left in the spectrum of the musical instrument's note?

Answer: Since the spectrum of a square wave only contains odd harmonics, the 5th line from the left in the square wave's spectrum is its 9th harmonic, which has a frequency of 1,440 Hz. Therefore the fundamental frequency of the square wave is given by 1,440 Hz divided by 9 i.e. 160 Hz. The 9th line from the left in the square wave's spectrum is its 17th harmonic, so its frequency is equal to 160 Hz times 17 i.e. 2,720 Hz. The 8th line from the left in the musical instrument's note's spectrum is its 8th harmonic, so the fundamental frequency of its note is equal to 2,720 Hz divided by 8 i.e. 340 Hz. Since the 4th and 15th lines in the spectrum of the musical instrument's note are its 4th and 15th harmonics respectively, their frequencies are equal to 340 Hz times 4 i.e. 1,360 Hz and 340 Hz times 15 i.e. 5,100 Hz respectively.

Scientific Inquiry discussion points

1. The Pythagorean scale, said to be first defined by the Greek mathematician after whom it is named, was based on the ratios of just two intervals -the octave (2/1) and the fifth (3/2). Its simplicity of construction served as the basis of the music of civilisations such as ancient Greece and China. The Pythagorean scale's drawback was that the ratio of the third was complex (81/64) and deemed unsatisfactory by many. As the interval of the third became more important, proponents of the Just scale, in which the ratio of the third was 5/4 instead of 81/64, much preferred it to the Pythagorean scale, as ratios with small numbers were considered by

the Greeks to be more beautiful than ratios with large numbers. The proponents of the Pythagorean scale of course disagreed strongly. Here we see the objective scientific inquiry of Pythagoras coming into conflict with subjective aesthetic judgement. Can you think of other examples in which subjective judgements come into conflict with objective scientific inquiry?

There are many examples of objective scientific inquiry coming into conflict with subjective perception. For example, many ancient civilisations believed in a system in which the sun revolved around the earth, as this seems to be supported by our subjective observation of the sun's motion. However, more detailed study of the sun's motion showed this was untenable, and eventually astronomers could explain it's actual motion only by adopting the theory that the earth revolves around the sun. A more recent example is the subjective belief that some races are superior to other races, simply because of factors such as the colour of their skin or other physical features. Modern understanding of genetics has shown that there is no actual scientific objective basis for the notion of racial superiority.