Answers to Tutorial No 2, Semester 2, 2024/25

1. You place your finger 18 cm from one end of a vibrating string which is 72 cm long and the string vibrates at a frequency of 540 Hz. Calculate the fundamental frequency of the string. If the string's length is increased by 25%, what is the distance your finger should be placed from the nearer end of the string so that the string will vibrate with a frequency of 648 Hz?

   Answer: Since 18 cm is one-quarter of the string's length of 72 cm, the string must be vibrating at its 4th harmonic frequency. Therefore the fundamental frequency of the string is given by 540 Hz divided by 4 i.e. 135 Hz. When the length of the string is increased by 25%, its length will be equal to 72 cm times 1.25 i.e. 90 cm. The fundamental frequency of the string will then be equal to 135 Hz times $\frac{72}{90}$ i.e. 108 Hz. Since 648 Hz divided by 108 Hz is 6, when the string vibrates with a frequency of 648 Hz, it is vibrating at its 6th harmonic. Therefore your finger should be placed at a distance equal to one-sixth of 90 cm i.e. 15 cm from the nearer end.

2. A string 80 cm long vibrates at a frequency of 1,200 Hz with 8 antinodes between its two ends. A second string vibrates at a frequency of 1,500 Hz with 5 antinodes between its two ends. What is the length of the second string? If a third string of length 60

cm vibrates at a frequency of 800 Hz, what is the number of nodes which this third string has between its two ends (not counting the nodes at both ends)? (Assume that the three strings are similar in all respects except for length.)

Answer: Since the first string has 8 antinodes it is at its 8th harmonic, and its fundamental frequency is equal to 1,200 Hz divided by 8 i.e. 150 Hz. The second string has 5 antinodes so it must be at its 5th harmonic, and its fundamental frequency is equal to 1,500 Hz divided by 5 i.e. 300 Hz. The length of the second string is hence given by 80 cm times $\frac{150}{300}$ i.e. 40 cm. The third string is 60 cm long, so its fundamental frequency is given by 150 Hz times $\frac{80}{60}$ i.e. 200 Hz. 800 Hz divided by 200 Hz is equal to 4, so the third string must be vibrating at its 4th harmonic and thus must have 4 antinodes and 3 nodes between its two ends (not counting the nodes at both ends).

3. We start from a first musical note and then go up by the interval of a Just sixth to arrive at a second note. We start again from the same first note and go up again, but this time by the interval of a Pythagorean sixth to arrive at a third note. Which of these two notes i.e. the second and third notes, has the higher frequency, and what is the ratio of the interval between these two notes? If the frequency of the first note is 150 Hz, calculate the frequencies of the second and third notes. If we go down instead of up from 150 Hz by the same two intervals i.e. the

Just sixth and the Pythagorean sixth, calculate the frequencies of the second and third notes.

Answer: A Just sixth has the ratio of $\frac{5}{3}$ which is approximately equal to 1.666 and the ratio of a Pythagorean sixth is $\frac{27}{16}$ which is equal to 1.6875, so the third note has a higher frequency than the second note. If the frequency of the first note is 150 Hz, the frequency of the second note is given by 150 Hz times $\frac{5}{3}$ i.e. 250 Hz, and the frequency of the third note is equal to 150 Hz times $\frac{27}{16}$ i.e. 253.125 Hz. The interval between these two notes has a ratio equal to $\frac{27}{16}$ divided by $\frac{5}{3}$ which is the same as $\frac{27}{16}$ multiplied by $\frac{3}{5}$ i.e. $\frac{81}{80}$. Going down instead of up, the frequency of the second note would now be equal to 150 Hz divided by $\frac{5}{3}$ which is the same as 150 Hz times $\frac{3}{5}$ i.e. 90 Hz, and the frequency of the third note would now be equal to 150 Hz times $\frac{16}{27}$ i.e. approximately 88.889 Hz.

4. The common pentatonic scale on a piano keyboard often used in the folk songs of many musical cultures can be found by playing only the black notes on the keyboard in sequence. This scale is called "pentatonic" which means "five notes" because it consists of only five notes (not counting the note one octave above the beginning of the scale). The common pentatonic scale has the following sequence of intervals: tone, tone, three semitones, tone, three semitones, arriving at the final note exactly one octave or 12 semitones above the starting note. Another type of pentatonic scale is the Balinese game-

lan pentatonic scale which has a different sequence of intervals: semitone, tone, 2 tones, semitone, 2 tones, making up a total of 12 semitones. Starting from the note D just above Middle C, give the letter names of the notes making up these two different pentatonic scales. Starting instead from the G just below Middle C, give the names of the notes making up these two pentatonic scales.

Answer: Starting from the note D, the next note in the common pentatonic scale is a tone above i.e. the note E, and the next note is also a tone above i.e. the note Fsharp/Gflat. Going up three semitones brings us to the next note A, and up another tone brings us to the note B. Up another three semitones brings us back to the note D one octave above the starting D. If we start from the first D again, the next note in the Balinese pentatonic scale one semitone up is Dsharp/Eflat, and going up by a tone gives us F. 2 tones up gives us A and another semitone up gives us Asharp/Bflat and up by 2 tones arrives at the D one octave above the starting D. Starting from the note G instead of D, the common pentatonic scale gives the notes G, A, B, D, E and G again. The Balinese pentatonic scale starting from G gives us G, Gsharp/Aflat, Asharp/Bflat, D, Dsharp/Eflat and G again.

5. The strings of a viola are tuned in Just fifths as is usual for a viola, and the viola's A string is tuned to a frequency of 440 Hz. A guitar's six strings are tuned relative to each other as is usual for a guitar, and its

A string is tuned to a frequency of 110 Hz. What are the frequencies of the viola's G string and its D string and the ratio of the interval between these two frequencies? What are the frequencies of the two musical notes on the guitar which are equivalent to these two notes on the viola, and what is the ratio between these two notes on the guitar? Calculate the ratio of the interval between the frequencies of the guitar's B3 note and the viola's D4 note. (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

Answer: Since the viola's G string which is the note G3 is two Just fifths (of which the ratio is $\frac{3}{2}$) below the viola's A string, the G string's frequency is given by 440 Hz divided by $\frac{3}{2}$ two times, which is the same as multiplying 440 Hz by $\frac{4}{9}$ i.e. approximately 195.556 Hz. Its D string which is the note D4 is one Just fifth below 440 Hz so its frequency is given by multiplying 440 Hz by $\frac{2}{3}$ i.e. approximately 293.333 Hz. The ratio between the frequencies of the viola's G and D strings is a Just fifth or $\frac{3}{2}$ or 1.5. Since the semitones on the guitar are all Equal-tempered, they have a ratio of approximately 1.05946. The guitar's A string is the note A2 with frequency 110 Hz, and its A3 note's frequency is double this i.e. 220 Hz. The guitar's G3 note is two semitones below its A3 note, and its frequency is thus equal to 220 Hz divided by 1.05946 two times, i.e. approximately 195.999 Hz. The guitar's D4 note is 5 semitones above its A3 note, so its frequency is approximately equal to 220 Hz multiplied by 1.05946 five times i.e.

approximately 293.660 Hz. The ratio between the guitar's G3 note and its D4 note is thus given by the ratio of seven Equal-tempered semitones i.e. approximately 1.4983. The guitar's B3 note being two semitones above A3 has a frequency given by 220 Hz times 1.05946 twice i.e. approximately 246.940 Hz, and the viola's D string which is also its D4 note has a frequency of approximately 293.333 Hz. Therefore the ratio between these two notes is approximately given by 293.333 Hz divided by 246.940 Hz i.e. approximately 1.1879.

6. The frequency spectrum of a musical note is represented by a graph with vertical lines along the x-axis. The positions of the lines on the x-axis represent the frequencies of the harmonics and the lengths of the lines represent the amplitudes of the harmonics. A musical wind instrument recently unearthed by archeologists is made to play a note which has a spectrum showing its fundamental frequency and all its harmonics up to the 21st harmonic. All harmonics (odd and even) are present in this spectrum. The 8th line from the left in this spectrum has the same frequency as the 7th line from the left in the spectrum of a square wave. If the frequency of the 4th line in the spectrum of the square wave is 1,400 Hz, calculate the frequencies of the 9th and 12th lines from the left in the spectrum of the musical instrument's note.

   Answer: The spectrum of a square wave only contains odd harmonics, so the 4th line from the left in its spectrum is its 7th harmonic which has a frequency of 1,400 Hz. The fundamental frequency of the square wave is thus given by 1,400 Hz divided by 7 i.e. 200 Hz. The 7th line from the left in the square wave's spectrum is its 13th harmonic, so its frequency is equal to 200 Hz times 13 i.e. 2,600 Hz. Since the 8th line from the left in the musical instrument's note's spectrum is its 8th harmonic, the fundamental frequency of its note is given by 2,600 Hz divided by 8 i.e. 325 Hz. The 9th and 12th lines in the spectrum of the musical instrument's note are its 9th and 12th harmonics respectively, so their frequencies are given by 325 Hz times 9 i.e. 2,925 Hz and 325 Hz times 12 i.e. 3,900 Hz respectively.

Scientific Inquiry discussion points

1. The Pythagorean scale, said to be first defined by the Greek mathematician after whom it is named, was based on the ratios of just two intervals -the octave (2/1) and the fifth (3/2). Its simplicity of construction served as the basis of the music of civilisations such as ancient Greece and China. The Pythagorean scale's drawback was that the ratio of the third was complex (81/64) and deemed unsatisfactory by many. As the interval of the third became more important, proponents of the Just scale, in which the ratio of the third was 5/4 instead of 81/64, much preferred it to the Pythagorean scale, as ratios with small numbers were considered by the Greeks to be more beautiful than ratios with large numbers. The proponents of the Pythagorean scale of course disagreed strongly. Here we see the objec-

tive scientific inquiry of Pythagoras coming into conflict with subjective aesthetic judgement. Can you think of other examples in which subjective judgements come into conflict with objective scientific inquiry?

There are many examples of objective scientific inquiry coming into conflict with subjective perception. For example, many ancient civilisations believed in a system in which the sun revolved around the earth, as this seems to be supported by our subjective observation of the sun's motion. However, more detailed study of the sun's motion showed this was untenable, and eventually astronomers could explain it's actual motion only by adopting the theory that the earth revolves around the sun. A more recent example is the subjective belief that some races are superior to other races, simply because of factors such as the colour of their skin or other physical features. Modern understanding of genetics has shown that there is no actual scientific objective basis for the notion of racial superiority.