

Answers to Tutorial No 2, Semester 2, 2025/26

1. If a 90 cm long string vibrates at a frequency of 750 Hz when you place your finger 30 cm from one end of the string, what is the fundamental frequency of the string? If the string's length is increased by 25%, what is the distance your finger should be placed from the nearer end of the string to enable the string to vibrate with a frequency of 1,200 Hz?

Answer: Since 30 cm is one-third of the string's length of 90 cm, the string is vibrating at its 3rd harmonic frequency. Therefore the fundamental frequency of the string is equal to 750 Hz divided by 3 i.e. 250 Hz. When the length of the string is increased by 25%, its length will be given by 90 cm times 1.25 i.e. 112.5 cm. The fundamental frequency of the string will then be equal to 250 Hz times $\frac{90}{112.5}$ i.e. 200 Hz. 1,200 Hz divided by 200 Hz is 6, so when the string vibrates with a frequency of 1,200 Hz, it is vibrating at its 6th harmonic. Therefore your finger should be placed at a distance equal to one-sixth of 112.5 cm i.e. 18.75 cm from the nearer end.

2. A string vibrating at a frequency of 1,600 Hz with 8 antinodes between its two ends is 80 cm long, and a second string is vibrating at a frequency of 1,250 Hz with 5 antinodes between its two ends. Calculate the length of the second string. A third string vibrating at a frequency of 1,120 Hz is 100 cm long. What

is the number of nodes which this third string has between its two ends (not counting the nodes at both ends)? (Assume that the three strings are similar in all respects except for length.)

Answer: Since the first string has 8 antinodes it is at its 8th harmonic, and its fundamental frequency is thus equal to 1,600 Hz divided by 8 i.e. 200 Hz. The second string has 5 antinodes so it is at its 5th harmonic, and its fundamental frequency is given by 1,250 Hz divided by 5 i.e. 250 Hz. Therefore the length of the second string is given by 80 cm times $\frac{200}{250}$ i.e. 64 cm. The third string is 100 cm long, so its fundamental frequency is given by 200 Hz times $\frac{80}{100}$ i.e. 160 Hz. Since 1,120 Hz divided by 160 Hz is equal to 7, the third string must be vibrating at its 7th harmonic and therefore has 7 antinodes and 6 nodes between its two ends (not counting the nodes at both ends).

3. If we start from a first musical note and then go upwards by the interval of a Just sixth, we arrive at a second note. If we start again from the same first note and go up again, but this time by the interval of a Pythagorean sixth, we will arrive at a third note. Which of these two notes i.e. the second and the third notes, has the higher frequency? Calculate the ratio of the interval between these two notes. If the frequency of the first note is 150 Hz, what are the frequencies of the second and third notes? If we had gone down instead of up from 150 Hz by the same two intervals i.e. the Just sixth and the Pythagorean

sixth, what would be the frequencies of the second and third notes be?

Answer: A Just sixth has the ratio of $\frac{5}{3}$ which is approximately equal to 1.6667 and the ratio of a Pythagorean sixth is $\frac{27}{16}$ which is equal to 1.6875, so the third note has a higher frequency than the second note. Therefore the interval between these two notes has a ratio equal to $\frac{27}{16}$ divided by $\frac{5}{3}$ which is the same as $\frac{27}{16}$ multiplied by $\frac{3}{5}$ i.e. $\frac{81}{80}$. If the frequency of the first note is 150 Hz, the frequency of the second note is equal to 150 Hz times $\frac{5}{3}$ i.e. 250 Hz, and the frequency of the third note is equal to 150 Hz times $\frac{27}{16}$ i.e. 253.125 Hz. If we go down instead of up, the frequency of the second note would now be equal to 150 Hz divided by $\frac{5}{3}$ which is the same as 150 Hz times $\frac{3}{5}$ i.e. 90 Hz, and the frequency of the third note would now be equal to 150 Hz times $\frac{16}{27}$ i.e. approximately 88.8889 Hz.

4. Playing only the black notes on the piano keyboard in sequence gives us the well-known common pentatonic scale often used in the folk songs of many musical cultures. Because it consists of only five notes (not counting the note one octave above the beginning of the scale, this scale is called “pentatonic” which means “five notes”. This common pentatonic scale has the following sequence of intervals: tone, tone, three semitones, tone, three semitones, arriving at the final note exactly one octave or 12 semitones above the starting note. Another type of pentatonic scale is the Balinese gamelan pentatonic scale which

has a different sequence of intervals: semitone, tone, 2 tones, semitone, 2 tones, making up a total of 12 semitones. Starting from Middle C, give the letter names of the notes making up these two different pentatonic scales. Starting instead from the F just below Middle C, give the names of the notes making up these two pentatonic scales.

Answer: Starting from Middle C, the next note in the common pentatonic scale is a tone above i.e. the note D, and the next note is also a tone above i.e. the note E. Going up three semitones brings us to the next note G, and up another tone brings us to the note A. Up another three semitones brings us back to the note C one octave above Middle C. Starting from Middle C again, the next note in the Balinese pentatonic scale one semitone up is Csharp/Dflat, and going up by a tone gives us Dsharp/Eflat. 2 tones up gives us G and another semitone up gives us Gsharp/Aflat and up by 2 tones arrives at the C one octave above Middle C. Starting from the note F instead of Middle C, the common pentatonic scale gives the notes F, G, A, C, D and F again. The Balinese pentatonic scale starting from F gives us F, Fsharp/Gflat, Gsharp/Aflat, Csharp/Dflat and F again.

5. The strings of a violin are tuned in Just fifths as is normal for a violin, with its A string tuned to a frequency of 440 Hz. The six strings of a guitar are tuned relative to each other as is normal for a guitar, with its A string at a frequency of 110 Hz. Calcu-

late the frequencies of the violin's G string and its E string and the ratio of the interval between these two frequencies. What are the frequencies of the two musical notes on the guitar which are the same as these two notes on the violin, and what is the ratio between these two notes on the guitar? Calculate the ratio of the interval between the frequencies of the guitar's G3 note and the violin's E5 note. (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

Answer: The violin's G string is the note G3 two Just fifths below the violin's A string, so the G string's frequency is equal to 440 Hz divided by $\frac{3}{2}$ twice, which is the same as multiplying 440 Hz by $\frac{4}{9}$ i.e. approximately 195.556 Hz. The violin's E string is the note E5 one Just fifth above the violin's A string, so the E string's frequency is equal to 440 Hz multiplied by $\frac{3}{2}$, i.e. 660 Hz. Therefore the ratio between the frequencies of the violin's G and E strings is three Just fifths or $\frac{3}{2}$ times $\frac{3}{2}$ times $\frac{3}{2}$ or $\frac{27}{8}$ i.e. 3.375. The semitones on the guitar are all Equal-tempered having a ratio of approximately 1.05946. The guitar's A string is the note A2 with frequency 110 Hz, and its A3 note's frequency is double this i.e. 220 Hz. The guitar's G3 note is two semitones below its A3 note, and its frequency is thus equal to 220 Hz divided by 1.05946 two times, i.e. approximately 195.999 Hz. The guitar's A5 note is three octaves above its A2 note i.e. it is 110 Hz times 2 three times, i.e. 880 Hz. The guitar's E5 note is 5 semitones below its A5 note, so its frequency is approximately equal to 880

Hz divided by 1.05946 five times i.e. approximately 659.2647 Hz. Hence the ratio between the guitar's G3 and E5 notes is given by 659.2647 Hz divided by 195.9989 Hz i.e. 3.364. The ratio between the guitar's G3 note and the violin's E5 note is given by 660 Hz divided by 195.9989 Hz i.e. 3.367.

6. The frequency spectrum of a musical note is represented by a graph with vertical lines along the x-axis, where the positions of the lines on the x-axis represent the frequencies of the harmonics and the lengths of the lines represent the amplitudes of the harmonics. A note sounded by an ancient wind instrument has a spectrum showing its fundamental frequency and all its harmonics (odd and even) up to the 21st harmonic. The 8th line from the left in this spectrum has the same frequency as the 6th line from the left in the spectrum of a square wave. If the frequency of the 5th line in the spectrum of the square wave is 2,700 Hz, calculate the frequencies of the 4th and 15th lines from the left in the spectrum of the musical instrument's note.

Answer: The spectrum of a square wave only contains odd harmonics, so the 5th line from the left in its spectrum is its 9th harmonic which has a frequency of 2,700 Hz. The fundamental frequency of the square wave is thus equal to 2,700 Hz divided by 9 i.e. 300 Hz. The 6th line from the left in the square wave's spectrum is its 11th harmonic, so its frequency is equal to 300 Hz times 11 i.e. 3,300 Hz. The 8th line from the left in the musical instrument's

note's spectrum is its 8th harmonic, so the fundamental frequency of its note is equal to 3,300 Hz divided by 8 i.e. 412.5 Hz. The 4th and 15th lines in the spectrum of the musical instrument's note are its 4th and 15th harmonics respectively, whose frequencies are given by 412.5 Hz times 4 i.e. 1,650 Hz and 412.5 Hz times 15 i.e. 6,187.5 Hz respectively.

Scientific Inquiry discussion points

1. The Pythagorean scale, said to be first defined by the Greek mathematician after whom it is named, was based on the ratios of just two intervals -the octave ($2/1$) and the fifth ($3/2$). Its simplicity of construction served as the basis of the music of civilisations such as ancient Greece and China. The Pythagorean scale's drawback was that the ratio of the third was complex ($81/64$) and deemed unsatisfactory by many. As the interval of the third became more important, proponents of the Just scale, in which the ratio of the third was $5/4$ instead of $81/64$, much preferred it to the Pythagorean scale, as ratios with small numbers were considered by the Greeks to be more beautiful than ratios with large numbers. The proponents of the Pythagorean scale of course disagreed strongly. Here we see the objective scientific inquiry of Pythagoras coming into conflict with subjective aesthetic judgement. Can you think of other examples in which subjective judgements come into conflict with objective scientific inquiry?

There are many examples of objective scientific inquiry coming into conflict with subjective perception. For example, many ancient civilisations believed in a sys-

tem in which the sun revolved around the earth, as this seems to be supported by our subjective observation of the sun's motion. However, more detailed study of the sun's motion showed this was untenable, and eventually astronomers could explain it's actual motion only by adopting the theory that the earth revolves around the sun. A more recent example is the subjective belief that some races are superior to other races, simply because of factors such as the colour of their skin or other physical features. Modern understanding of genetics has shown that there is no actual scientific objective basis for the notion of racial superiority.