

Answers to Tutorial No 2, Semester 1, 2026/27

1. When you place your finger 25 cm from one end of a vibrating string 150 cm long, it vibrates at a frequency of 540 Hz. Calculate the fundamental frequency of the string. If the string's length is increased by 20%, calculate the distance your finger should be placed from the nearer end of the string to enable the string to vibrate with a frequency of 600 Hz

Answer: 25 cm is one-sixth of the string's length of 150 cm, so the string is vibrating at its 6th harmonic frequency. The fundamental frequency of the string is thus equal to 540 Hz divided by 6 i.e. 90 Hz. On increasing the length of the string by 20%, its length will be equal to 150 cm times 1.2 i.e. 180 cm. Therefore the fundamental frequency of the string will be equal to 90 Hz times $\frac{150}{180}$ i.e. 75 Hz. 600 Hz divided by 75 Hz is 8, so the string is vibrating at its 8th harmonic. Therefore your finger should be placed at a distance equal to one-eighth of 180 cm i.e. 22.5 cm from the nearer end.

2. A string 60 cm long is vibrating at a frequency of 1,200 Hz with 6 antinodes between its two ends. A second string is vibrating at a frequency of 400 Hz with 4 antinodes between its two ends. What is the length of the second string? A third string vibrating at a frequency of 360 Hz is 100 cm long. Calculate

the number of nodes which this third string has between its two ends (not counting the nodes at both ends). (Assume that the three strings are similar in all respects except for length.)

Answer: The first string has 6 antinodes so it is at its 6th harmonic. Therefore its fundamental frequency is given by 1,200 Hz divided by 6 i.e. 200 Hz. Since the second string has 4 antinodes, it is at its 4th harmonic, and its fundamental frequency is equal to 400 Hz divided by 4 i.e. 100 Hz. The length of the second string is thus equal to 60 cm times $\frac{200}{100}$ i.e. 120 cm. Since the third string is 100 cm long, its fundamental frequency is given by 100 Hz times $\frac{120}{100}$ i.e. 120 Hz. 360 Hz divided by 120 Hz is equal to 3, so the third string is vibrating at its 3rd harmonic. The third string therefore has 3 antinodes and 2 nodes between its two ends (not counting the nodes at both ends).

3. Starting from a first musical note and then going upwards by the interval of a Pythagorean third, we arrive at a second note. Starting again from the same first note and going up again, but this time by the interval of a Just third, we arrive at a third note. Of these two notes i.e. the second and the third notes, which one has the higher frequency? What is the ratio of the interval between these two notes? If the frequency of the first note is 120 Hz, what are the frequencies of the second and third notes? If we had gone down instead of up from 120 Hz by the same two intervals i.e. the Pythagorean third and

the Just third, calculate the frequencies of the second and third notes in this case.

Answer: Since a Pythagorean third has the ratio of $\frac{81}{64}$ which is equal to 1.265625 and the ratio of a Just third is $\frac{5}{4}$ which is equal to 1.25, the second note has a higher frequency than the third note. The interval between these two notes has a ratio given by $\frac{81}{64}$ divided by $\frac{5}{4}$ which is the same as $\frac{81}{64}$ multiplied by $\frac{4}{5}$ i.e. $\frac{81}{80}$. If the frequency of the first note is 120 Hz, the frequency of the second note is equal to 120 Hz times $\frac{81}{64}$ i.e. 151.875 Hz, and the frequency of the third note is equal to 120 Hz times $\frac{5}{4}$ i.e. 150 Hz. Going down instead of up, the frequency of the second note would then be equal to 120 Hz divided by $\frac{81}{64}$ which is the same as 120 Hz times $\frac{64}{81}$ i.e. approximately 94.8148 Hz, and the frequency of the third note would be equal to 120 Hz times $\frac{4}{5}$ i.e. 96 Hz.

4. If we play only the black notes on the piano keyboard in sequence, this results in the well-known common pentatonic scale which is used in the folk songs of many countries. This scale consists of only five notes (not counting the note one octave above the beginning of the scale), so it is called “pentatonic” which means “five notes”. This is the sequence of intervals in the common pentatonic scale: tone, tone, three semitones, tone, three semitones, arriving at the final note exactly one octave or 12 semitones above the starting note. A different type of pentatonic scale is the Balinese gamelan pentatonic scale. This scale has a different sequence of intervals: semitone, tone,

2 tones, semitone, 2 tones, which makes a total of 12 semitones. If we start from the D just above Middle C, give the letter names of the notes making up these two different pentatonic scales. If we start instead from the A just below Middle C, give the names of the notes making up these two pentatonic scales.

Answer: Starting from D, the next note in the common pentatonic scale is a tone above i.e. the note E, and the next note is also a tone above i.e. the note F sharp. If we go up three semitones this brings us to the next note A, and up another tone brings us to the note B. Another three semitones up brings us back to the note D one octave above the starting D. Starting from D above Middle C again, the next note in the Balinese pentatonic scale one semitone up is Dsharp/Eflat, and going up by a tone gives us F. 2 tones up gives us A and another semitone up gives us Asharp/Bflat and up by 2 tones arrives at the D just above the C one octave above Middle C. Starting from the note A just below Middle C, the common pentatonic scale gives the notes A, B, Csharp, E, Fsharp and A again. The Balinese pentatonic scale starting from A gives us A, Asharp/Bflat, C, E, F and A again.

5. The six strings of a guitar are tuned relative to each other as is usual for a guitar, with its A string at a frequency of 110 Hz. The strings of a viola are tuned in Just fifths as is usual for a viola, with its A string tuned to a frequency of 440 Hz. What are the fre-

quencies of the viola's C string and its D string and the ratio of the interval between these two frequencies? What are the frequencies of the two musical notes on the guitar which are the same as these two notes on the viola? What is the ratio between these two notes on the guitar? Calculate the ratio of the interval between the frequencies of the guitar's C3 note and the viola's A4 note. (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

Answer: Since the viola's C string is the note C3 three Just fifths below the viola's A string, the C string's frequency is equal to 440 Hz divided by $\frac{3}{2}$ three times, which is the same as multiplying 440 Hz by $\frac{8}{27}$ i.e. approximately 130.370 Hz. The viola's D string is the note D4 one Just fifth below the viola's A string, so the D string's frequency is equal to 440 Hz multiplied by $\frac{2}{3}$, i.e. approximately 293.333 Hz. The ratio between the frequencies of the viola's C and D strings is two Just fifths or $\frac{3}{2}$ times $\frac{3}{2}$ or $\frac{9}{4}$ i.e. 2.25. The guitar's A string is the note A2 with frequency 110 Hz, and its A3 note's frequency is double this i.e. 220 Hz. Since the semitones on the guitar are all Equal-tempered having a ratio of approximately 1.05946, the guitar's C3 note is three semitones above its A2 note, and its frequency is thus equal to 220 Hz multiplied by 1.05946 three times, i.e. approximately 130.8116 Hz. The guitar's D4 note is 5 semitones above its A3 note, so its frequency is approximately equal to 220 Hz multiplied by 1.05946 five times i.e. approximately 293.660 Hz.

The ratio between the guitar's D4 and C3 notes is given by an interval of 14 semitones or an octave and two semitones i.e. approximately 2 multiplied by 1.05946 two times or 2.245. The ratio between the guitar's C3 note and the viola's A4 note is given by 440 Hz divided by 130.8116 Hz i.e. approximately 3.367.

6. The frequency spectrum of a musical note is represented by a graph with vertical lines along the x-axis, where the positions of the lines on the x-axis represent the frequencies of the harmonics and the lengths of the lines represent the amplitudes of the harmonics. A note sounded by a newly invented wind instrument has a spectrum showing its fundamental frequency and all its harmonics (odd and even) up to the 21st harmonic. The 6th line from the left in this spectrum has the same frequency as the 9th line from the left in the spectrum of a square wave. If the frequency of the 7th line in the spectrum of the square wave is 3,978 Hz, what are the frequencies of the 3rd and 13th lines from the left in the spectrum of the musical instrument's note?

Answer: Since the spectrum of a square wave only contains odd harmonics, the 7th line from the left in its spectrum is its 13th harmonic which has a frequency of 3,978 Hz. The fundamental frequency of the square wave is thus equal to 3,978 Hz divided by 13 i.e. 306 Hz. The 9th line from the left in the square wave's spectrum is its 17th harmonic, so its frequency is equal to 306 Hz times 17 i.e. 5,202

Hz. The 6th line from the left in the musical instrument's note's spectrum is its 6th harmonic, so the fundamental frequency of its note is equal to 5,202 Hz divided by 6 i.e. 867 Hz. The 3rd and 13th lines in the spectrum of the musical instrument's note are its 3rd and 13th harmonics respectively. Their frequencies are given by 867 Hz times 3 i.e. 2,601 Hz and 867 Hz times 13 i.e. 11,271 Hz respectively.

Scientific Inquiry discussion points

1. The Pythagorean scale, said to be first defined by the Greek mathematician after whom it is named, was based on the ratios of just two intervals -the octave ($2/1$) and the fifth ($3/2$). Its simplicity of construction served as the basis of the music of civilisations such as ancient Greece and China. The Pythagorean scale's drawback was that the ratio of the third was complex ($81/64$) and deemed unsatisfactory by many. As the interval of the third became more important, proponents of the Just scale, in which the ratio of the third was $5/4$ instead of $81/64$, much preferred it to the Pythagorean scale, as ratios with small numbers were considered by the Greeks to be more beautiful than ratios with large numbers. The proponents of the Pythagorean scale of course disagreed strongly. Here we see the objective scientific inquiry of Pythagoras coming into conflict with subjective aesthetic judgement. Can you think of other examples in which subjective judgements come into conflict with objective scientific inquiry?

There are many examples of objective scientific inquiry coming into conflict with subjective perception. For

example, many ancient civilisations believed in a system in which the sun revolved around the earth, as this seems to be supported by our subjective observation of the sun's motion. However, more detailed study of the sun's motion showed this was untenable, and eventually astronomers could explain it's actual motion only by adopting the theory that the earth revolves around the sun. A more recent example is the subjective belief that some races are superior to other races, simply because of factors such as the colour of their skin or other physical features. Modern understanding of genetics has shown that there is no actual scientific objective basis for the notion of racial superiority.