Tutorial No 2, Semester 2, 2023/24

1. When you place your finger 18 cm from one end of a string which is 54 cm long, it vibrates at a frequency of 360 Hz. Calculate the fundamental frequency of the string. If the string's length is then increased by 25%, how far should your finger be placed from the nearer end of the string so that the string will vibrate with a frequency of 480 Hz?

   Answer: Since 18 cm is one-third of the string's length of 54 cm, the string must be vibrating at its 3rd harmonic frequency, which means that the fundamental frequency of the string is equal to 360 Hz divided by 3 i.e. 120 Hz. When the length of the string is increased by 25%, its length will be given by 54 cm times 1.25 i.e. 67.5 cm. Therefore the fundamental frequency of the string will be given by 120 Hz times $\frac{54}{67.5}$ i.e. 96 Hz. When the string vibrates with a frequency of 480 Hz, it must be vibrating at its 5th harmonic since 480 Hz divided by 96 Hz is 5. Your finger should thus be placed at a distance equal to one-fifth of 67.5 cm i.e. 13.5 cm from the nearer end.

2. A string which is 75 cm long is vibrating with 5 antinodes between its two ends at a frequency of 1,500 Hz. A second string is vibrating at a frequency of 900 Hz with 6 antinodes between its two ends. What is the length of the second string? If a third string of length

90 cm is vibrating at a frequency of 2,250 Hz, calculate the number nodes which this third string has between its two ends (not counting the nodes at both ends). (Assume that the three strings are similar in all respects except for length.)

3. If we start from a first musical note and then go up by the interval of a Just sixth, we will arrive at a second note. If we start again from the same first note, but now go up again this time by the interval of a Pythagorean sixth, we will arrive at a third note. Which of these two notes i.e. the second and third notes, has the higher frequency, and what is the ratio of the interval between these two notes? If the frequency of the first note is 120 Hz, what are the frequencies of the second and third notes? If we start again from the same first note of frequency 120 Hz, but now go down instead of up by the same two intervals i.e. the Just sixth and the Pythagorean sixth, calculate the frequencies of the second and third notes.

4. The common pentatonic scale often used in the folk songs of many musical cultures can be found on a piano by playing only the black notes on its keyboard in sequence. The term "pentatonic" (meaning "five notes") scale is so-called because the scale consists of only five notes (not counting the note one octave above the beginning of the scale). The common pentatonic scale is constructed using the following sequence of intervals: tone, tone, three semitones, tone, followed by three semitones, arriving at the

final note exactly one octave or 12 semitones above the starting note. A different type of pentatonic scale is the Balinese gamelan pentatonic scale which has a different sequence of intervals: semitone, tone, 2 tones, semitone, 2 tones, making up a total of 12 semitones. If we start from the note A just below Middle C, what are the letter names of the notes making up these two different pentatonic scales? If you start instead from the G just above Middle C, what are the names of the notes making up these two pentatonic scales?

5. All the strings of a 'cello are tuned in Just fifths as is normal for a 'cello, and its A string is tuned to a frequency of 220 Hz. A bass guitar's four strings are tuned relative to each other as usual for a bass guitar, and its A string is tuned to a frequency of 55 Hz. Calculate the frequencies of the 'cello's C string and its G string and the ratio of the interval between these two frequencies. Calculate also the frequencies on the bass guitar of the two musical notes which are equivalent to these two notes on the violin, and give the ratio between these two notes on the bass guitar. What is the ratio of the interval between the frequencies of the bass guitar's G2 note and the 'cello's G2 note? (Take the ratio of an Equal-tempered semitone to be equal to 1.05946 for your calculations.)

6. The graph of the spectrum of a musical sound has vertical lines on the x-axis which represent the fundamental frequency and harmonics of the sound. The lengths of the lines represent the amplitudes of the

harmonics, and their positions on the horizontal x-axis represent their frequencies. A newly invented musical wind instrument produces a note which has a spectrum showing its fundamental frequency and all its harmonics up to the 19th harmonic, and all harmonics (odd and even) are present in this spectrum. The 5th line from the left in this spectrum has the same frequency as the 8th line from the left in the spectrum of a square wave. If the frequency of the 6th line in the spectrum of the square wave is 1,320 Hz, calculate the frequencies of the 7th and 17th lines from the left in the spectrum of the musical instrument's note.

Scientific Inquiry discussion points

1. The Pythagorean scale, said to be first defined by the Greek mathematician after whom it is named, was based on the ratios of just two intervals -the octave (2/1) and the fifth (3/2). Its simplicity of construction served as the basis of the music of civilisations such as ancient Greece and China. The Pythagorean scale's drawback was that the ratio of the third was complex (81/64) and deemed unsatisfactory by many. As the interval of the third became more important, proponents of the Just scale, in which the ratio of the third was 5/4 instead of 81/64, much preferred it to the Pythagorean scale, as ratios with small numbers were considered by the Greeks to be more beautiful than ratios with large numbers. The proponents of the Pythagorean scale of course disagreed strongly. Here we see the objective scientific inquiry of Pythagoras coming into conflict

with subjective aesthetic judgement. Can you think of other examples in which subjective judgements come into conflict with objective scientific inquiry?