Answers to Tutorial No 3, Semester 1, 2024/25

1. A string which is vibrating with a frequency of 1,800 Hz has 9 antinodes between its two ends. A second string which has 6 nodes between its two ends (not counting the nodes at either end) is vibrating at a frequency of 1,050 Hz and is 60 cm long. Calculate the length of the first string which is vibrating with 9 antinodes. If a third string which is 80 cm long is vibrating at a frequency of 675 Hz, how many nodes would this third string have between its two ends (not counting the nodes at either end)? (Assume that the three strings are similar in all respects except length.)

   Answer: Since the first string has 9 antinodes it must be at its 9th harmonic and hence its fundamental frequency is equal to 1,800 Hz divided by 9 i.e. 200 Hz. The second string is vibrating with 6 nodes so it must have 7 antinodes and is vibrating at its 7th harmonic, so its fundamental frequency is given by 1,050 Hz divided by 7 i.e. 150 Hz. Therefore the length of the first string is given by 60 cm times $\frac{150}{200}$ i.e. 45 cm. The third string is 80 cm long so its fundamental frequency is equal to 150 Hz times $\frac{60}{80}$ i.e. 112.5 Hz. Since the third string is vibrating at a frequency of 675 Hz, it must be vibrating at its 6th harmonic as 675 Hz divided by 112.5 Hz is equal to 6. Hence the third string must have 6 antinodes and 5 nodes between its two ends (not counting the nodes at either end).

2. A kayak (small boat) is paddling along the surface of the sea in the same direction and the same speed as the water waves on the surface of the sea. The total length of the kayak is exactly equal to 5 complete wavelengths of the sea waves which are moving with a speed of 1 metre per second with a frequency of 1.25 Hz. What is the length of the kayak? The frequency of the waves then increases to 1.5 Hz and the speed of the waves increases to 1.5 metre per second. Calculate the number of wavelengths of the waves which would exactly equal the length of the kayak when this happens.
   Answer: Since the sea waves have a wavelength equal to 1 metre per second divided by 1.25 Hz i.e. 0.8 metres, the length of the kayak is equal to 0.8 metres times 5 i.e. 4 metres. When the frequency of the waves increases to 1.5 Hz and the speed of the waves increases to 1.5 metres per second, the wavelength of the waves is then equal to 1.5 metres per second divided by 1.5 Hz i.e. 1 metre. The number of wavelengths which would exactly equal the length of the kayak is hence given by 4 metres divided by 1 metre i.e. 4 wavelengths.

3. A string which has a fundamental frequency of 180 Hz is vibrating with 5 antinodes between its two ends, and its frequency is the same as that of a closed pipe of length p cm which is vibrating with 4 nodes between its two ends (not counting the node at one

end). Calculate the fundamental frequency of the closed pipe. When the closed pipe vibrates with 6 nodes between its two ends (not counting the node at one end), its frequency is the same as that of an open pipe vibrating with 4 antinodes between its two ends (not counting the antinodes at both ends). Calculate the length of the open pipe.

Answer: Since the string is vibrating with 5 antinodes it is at its 5th harmonic, and its frequency of vibration is equal to 180 Hz times 5 i.e. 900 Hz. The closed pipe has 4 nodes so it must be at its 9th harmonic and its fundamental frequency is equal to 900 Hz divided by 9 i.e. 100 Hz. When the closed pipe has 6 nodes, it will be at its 13th harmonic and its frequency of vibration will be equal to 100 Hz times 13 i.e. 1,300 Hz. The open pipe has 4 antinodes, so it will have 5 nodes and will be at its 5th harmonic and its fundamental frequency is given by 1,300 Hz divided by 5 i.e. 260 Hz. An open pipe which has the same length p cm as the closed pipe would have a fundamental frequency double that of the closed pipe i.e. 200 Hz. Therefore the open pipe which has a fundamental frequency of 260 Hz must have a length given by p cm times $\frac{200}{260}$ i.e. $\frac{10p}{13}$ cm.

4. A string which is vibrating with 5 nodes (not counting the nodes at both ends) is 30 cm long. The note produced by this string combines with a note from a closed pipe which has a fundamental frequency of 120 Hz to produce beats of 12 Hz. The closed pipe is vibrating with 3 nodes between its two ends (not

counting the node at one end). When the string is then slightly shortened, the beat frequency increases (without passing through 0 Hz). What is the fundamental frequency of the string? If the string is then shortened from 30 cm to 28.4 cm, and assuming that the beats are still produced by the same harmonics of the string and the closed pipe as before, calculate the new beat frequency. If the length of the closed pipe is increased to 120% of its original length, calculate what the beat frequency would then be, assuming that the string is still 32.7 cm long.

Answer: The closed pipe has 3 nodes, so it is at its 7th harmonic, and its frequency is hence equal to 120 Hz times 7 i.e. 840 Hz. On shortening the string slightly its frequency increases. If the beat frequency increases, the frequency of the string must have been higher than that of the closed pipe, and since the beat frequency is 12 Hz, the frequency of the string is equal to 840 Hz plus 12 Hz i.e. 852 Hz. Since the string has 5 nodes and 6 antinodes it must be at its 6th harmonic and its fundamental frequency is given by 852 Hz divided by 6 i.e. 142 Hz. Since the shortened string has a length of 28.4 cm, its fundamental frequency would be equal to 142 Hz times $\frac{30}{28.4}$ i.e. 150 Hz. The string's 6th harmonic would then be 150 Hz times 6 i.e. 900 Hz and the beat frequency would then change to 900 Hz minus 840 Hz i.e. 60 Hz. When the length of the closed pipe is increased to 120% of its original length, its fundamental frequency would change to 120 Hz times $\frac{1}{1.2}$ i.e. 100

Hz. Its 7th harmonic would then be equal to 100 Hz times 7 i.e. 700 Hz, and the beat frequency would change to 900 Hz minus 700 Hz i.e. 200 Hz.

5. An electronic tuner which is producing a musical note with a frequency of 220 Hz is used by a 'cellist in tuning her 'cello's A string. When the note from the A string combines with the note from the tuner, beats of 6 Hz are heard. When the 'cellist gradually tightens the A string of the 'cello, the beat frequency gradually decreases (without passing through 0 Hz) to 4 Hz. What was the frequency of the note produced by the 'cello's A string when the beat frequency was equal to 6 Hz? To make the frequency of the A string come as close as possible to 220 Hz, what should the 'cellist do? If the beat frequency had increased to 7 Hz instead of decreasing when the string was tightened, what would the A string's frequency have been when the beat frequency was 6 Hz?

   Answer: Since the beat frequency was 6 Hz, the frequency of the A string's note was either 220 Hz minus 6 Hz i.e. 214 Hz, or 220 Hz plus 6 Hz i.e. 226 Hz. When the 'cello's A string was tightened, its frequency would have increased, and since the beat frequency then decreased to 4 Hz, this meant that the frequency of the A string's note must have moved closer to 220 Hz, so the A string's frequency must have been lower than 220 Hz when the beat frequency was 6 Hz i.e. the A string's frequency must have been equal to 214 Hz. To bring the frequency of the A string closer to 220 Hz, the 'cellist should increase its frequency further by continuing to tighten the A string so that the beat frequency further decreases. When the beat frequency reaches zero, the frequency of the A string must then be equal to 220 Hz. If the beat frequency had decreased to 7 Hz on tightening the A string, the frequency of the 'cello's note must have been higher than 220 Hz i.e. it must have been 226 Hz when the beat frequency was 6 Hz.

Scientific Inquiry discussion points

The Equal-tempered scale is obtained by dividing an octave into twelve equal steps. This gives us a scale with twelve notes, which has become the basis of most of Western music, whether classical, popular, folk, rock or any other genre of music. The democratic equality of these twelve notes enables music to modulate into any of the twelve available keys with ease, using just twelve notes in one octave. This greatly simplifies the design of musical instruments and how they are played. But this is not a "perfect" system in mathematical terms, as the important interval of the fifth is not exactly 3/2 as in the Just and Pythagorean scales, but deviates by ever so slightly an amount which is not apparent to most listeners. Are there other examples in science and technology where imperfections are an important part of the basis of an actual working system?

While we may consider mathematical and physical perfection to be most desirable, in the real world most things and processes deviate from mathematical perfec-

tion. One crucial example is in the DNA of our genetic code. The reproduction of DNA as it replicates in the multiplication of living cells is not perfect, in that errors may occur in the replication due to natural events such as the alteration of the DNA code by natural radiation or cosmic rays. This may seem undesirable and it does lead to undesirable effects sometimes, but this same process makes evolution possible, as the errors in replication allow changes in the make-up of livings things, which may then make an organism less or more suited to the changing environment. Hence the progress brought about by evolution depends on these imperfections in replication.