# Answers to Tutorial No 3, Semester 2, 2024/25

1. A string which has 8 antinodes between its two ends is vibrating with a frequency of 1,680 Hz. A second string 42 cm long which has 6 nodes between its two ends (not counting the nodes at either end) is vibrating at a frequency of 1,260 Hz. What is the length of the first string which is vibrating with 8 antinodes? If a third string which is 60 cm long is vibrating at a frequency of 1,134 Hz, how many nodes would this third string have between its two ends (not counting the nodes at either end)? (Assume that the three strings are similar in all respects except length.)

   **Answer:** The first string has 8 antinodes so it must be at its 8th harmonic. Therefore its fundamental frequency is equal to 1,680 Hz divided by 8 i.e. 210 Hz. Since the second string is vibrating with 6 nodes, it must have 7 antinodes and is thus vibrating at its 7th harmonic. Its fundamental frequency is therefore equal to 1,260 Hz divided by 7 i.e. 180 Hz. The length of the first string is thus equal to 42 cm times $\frac{180}{210}$ i.e. 36 cm. The third string is 60 cm long so its fundamental frequency is given by 180 Hz times $\frac{42}{60}$ i.e. 126 Hz. Since the third string is vibrating at a frequency of 1,134 Hz, it must be vibrating at its 9th harmonic as 1,134 Hz divided by 126 Hz is equal to 9. Therefore the third string must have 9 antinodes and 8 nodes between its two ends (not counting the nodes at either end).

2. A sailing boat is moving along in the same direction and the same speed as the sea waves on the surface of the sea, and the total length of the boat is exactly equal to 6 complete wavelengths of the sea waves. If the waves are moving with a speed of 0.9 metre per second with a frequency of 1 Hz, what is the length of the boat? If the speed of the waves then decreases to 0.5 metre per second and the frequency decreases to 0.8 Hz, what is the number of wavelengths of the waves which would exactly equal the length of the boat?

   **Answer:** The sea waves have a wavelength equal to 0.9 metre per second divided by 1 Hz i.e. 0.9 metres, so the length of the boat is given by 0.9 metres times 6 i.e. 5.4 metres. When the frequency of the waves decreases to 0.8 Hz and the speed of the waves decreases to 0.5 metres per second, the wavelength of the waves is then equal to 0.5 metres per second divided by 0.8 Hz i.e. 0.625 metre. The number of wavelengths which would exactly equal the length of the sailing boat is hence given by 5.4 metres divided by 0.625 metres i.e. 8.64 wavelengths.

3. A string vibrating with 4 antinodes between its two ends has a fundamental frequency of 220 Hz. Its frequency is the same as that of a closed pipe of length $k$ cm which is vibrating with 5 nodes between its two ends (not counting the node at one end). What is the fundamental frequency of the closed pipe? When the closed pipe vibrates with 7 nodes between its two ends (not counting the node at one end), its fre-

quency is the same as that of an open pipe vibrating with 7 antinodes between its two ends (not counting the antinodes at both ends). What is the length of the open pipe?

**Answer:** The string is vibrating with 4 antinodes so it is at its 4th harmonic, so its frequency of vibration is equal to 220 Hz times 4 i.e. 880 Hz. Since the closed pipe has 5 nodes, it is at its 11th harmonic and its fundamental frequency is equal to 880 Hz divided by 11 i.e. 80 Hz. When the closed pipe has 7 nodes, it will be at its 15th harmonic, so its frequency of vibration will be given by 80 Hz times 15 i.e. 1,200 Hz. Since the open pipe has 7 antinodes it will have 8 nodes and will be at its 8th harmonic. Its fundamental frequency is therefore equal to 1,200 Hz divided by 8 i.e. 150 Hz. An open pipe with the same length $k$ cm as the closed pipe would have a fundamental frequency double that of the closed pipe i.e. 160 Hz, so the open pipe having a fundamental frequency of 150 Hz has a length equal to $k$ cm times $\frac{160}{150}$ i.e. $\frac{16k}{15}$ cm.

4. A 66 cm string is vibrating with 6 nodes (not counting the nodes at both ends). The musical note produced by this string combines with a closed pipe which has a fundamental frequency of 98 Hz and is vibrating with 4 nodes between its two ends (not counting the node at one end). Beats of 14 Hz are produced, and when the string is then shortened slightly, the beat frequency increases (without passing through 0 Hz). Calculate the fundamental fre-

quency of the string. If the string is shortened from 66 cm to 64 cm, and assuming that the beats are still produced by the same harmonics of the string and the closed pipe as before, what would be the new beat frequency? If the length of the closed pipe is decreased to 80% of its original length, what would the beat frequency then be, assuming that the string is still 64 cm long.

**Answer:** Since the closed pipe has 4 nodes it is at its 9th harmonic, and its frequency is therefore equal to 98 Hz times 9 i.e. 882 Hz. Since on shortening the string slightly the beat frequency increases, the frequency of the string must have been higher than that of the closed pipe. When the beat frequency is 14 Hz so the frequency of the string is equal to 882 Hz plus 14 Hz i.e. 896 Hz. The string has 6 nodes and 7 antinodes so it is at its 7th harmonic and its fundamental frequency is given by 896 Hz divided by 7 i.e. 128 Hz. The shortened string has a length of 64 cm, so its fundamental frequency would be equal to 128 Hz times $\frac{66}{64}$ i.e. 132 Hz. The string's 7th harmonic would then be 132 Hz times 7 i.e 924 Hz so the beat frequency would become 924 Hz minus 882 Hz i.e. 42 Hz. If the length of the closed pipe is decreased to 80% of its original length, its fundamental frequency would change to 98 Hz times $\frac{1}{0.8}$ i.e. 122.5 Hz. Its 9th harmonic would then be equal to 122.5 Hz times 9 i.e. 1,102.5 Hz, and the beat frequency would change to 1,102.5 Hz minus 924 Hz i.e. 178.5 Hz.

5. A viola player is tuning her viola's A string using an electronic tuner which is producing a musical note with a frequency of 440 Hz. Beats of 8 Hz are heard when the note from the A string combines with the note from the tuner. When the viola player gradually loosens the A string of the viola, the beat frequency gradually decreases (without passing through 0 Hz) to 6 Hz. What was the frequency of the note produced by the viola's A string when the beat frequency was equal to 8 Hz? What should the viola player do in order to make the frequency of the A string come as close as possible to 440 Hz? If the beat frequency had increased to 9 Hz instead of decreasing when the string was loosened, calculate the A string's frequency when the beat frequency was 8 Hz.

   **Answer:** The beat frequency was 8 Hz, so the frequency of the A string's note was either 440 Hz minus 8 Hz i.e. 432 Hz, or 440 Hz plus 8 Hz i.e. 448 Hz. When the viola's A string was loosened, its frequency would have decreased. Since the beat frequency then decreased to 6 Hz, the frequency of the A string's note must have moved closer to 440 Hz, meaning that the A string's frequency must have been higher than 440 Hz when the beat frequency was 8 Hz. Hence the A string's frequency must have been equal to 448 Hz, and to bring the frequency of the A string even closer to 440 Hz, the viola player should continue to loosen the A string so that the beat frequency further decreases. When the beat frequency reaches zero, the frequency of the A string must then be equal to 440

Hz. If the beat frequency had increased to 9 Hz on loosening the A string, the frequency of the viola's note must have been lower than 440 Hz. Therefore it must have been at 432 Hz when the beat frequency was 8 Hz.

6. A closed pipe labelled A which has a fundamental frequency $f$ Hz is sliced into nine pieces of equal lengths, creating eight short open pipes labelled A1 to A8, and a short closed pipe A9. Three of the short open pipes A6, A7 and A8 are joined up with A9 to make a closed pipe labelled B. The remaining short open pipes A1 to A5 are joined up to make an open pipe labelled C. What are the fundamental frequencies of the pipes A9, B and C? Calculate the ratio of the interval between the frequency of B when it vibrates with 6 nodes between its two ends (not counting the node at one end), and the frequency of C when it vibrates with 11 nodes between its two ends.

   **Answer:** The short closed pipe A9 is one-ninth the length of A, so its fundamental frequency is given by $9f$ Hz. Since the closed pipe B is four times the length of A9, its fundamental frequency will be equal to $\frac{9f}{4}$ Hz. When B has 6 nodes, it will be at its 13th harmonic. Therefore its frequency will be equal to $\frac{9f}{4}$ Hz times 13 i.e. $\frac{117f}{4}$ Hz. A closed pipe which has a length five times the length of A9 will have a fundamental frequency of $\frac{9f}{5}$ Hz, so C which is an open pipe of the same length will have a fundamental frequency double this i.e. $\frac{18f}{5}$ Hz. When C has 11 nodes it will be at its 11th harmonic and its frequency will then be given by $\frac{18f}{5}$ Hz times 11 i.e. $\frac{198f}{5}$ Hz. The ratio of the interval between $\frac{198f}{5}$ Hz and $\frac{117f}{4}$ Hz is given by $\frac{198f}{5}$ Hz divided by $\frac{117f}{4}$ Hz i.e. $\frac{88}{65}$.

## Scientific Inquiry discussion points

The Equal-tempered scale is obtained by dividing an octave into twelve equal steps. This gives us a scale with twelve notes, which has become the basis of most of Western music, whether classical, popular, folk, rock or any other genre of music. The democratic equality of these twelve notes enables music to modulate into any of the twelve available keys with ease, using just twelve notes in one octave. This greatly simplifies the design of musical instruments and how they are played. But this is not a "perfect" system in mathematical terms, as the important interval of the fifth is not exactly 3/2 as in the Just and Pythagorean scales, but deviates by ever so slightly an amount which is not apparent to most listeners. Are there other examples in science and technology where imperfections are an important part of the basis of an actual working system?

*While we may consider mathematical and physical perfection to be most desirable, in the real world most things and processes deviate from mathematical perfection. One crucial example is in the DNA of our genetic code. The reproduction of DNA as it replicates in the multiplication of living cells is not perfect, in that errors may occur in the replication due to natural events such* *as the alteration of the DNA code by natural radiation or cosmic rays. This may seem undesirable and it does lead to undesirable effects sometimes, but this same process makes evolution possible, as the errors in replication allow changes in the make-up of livings things, which may then make an organism less or more suited to the changing environment. Hence the progress brought about by evolution depends on these imperfections in replication.*