Answers to Tutorial No 3, Semester 2, 2025/26

1. A string is vibrating with 7 antinodes between its two ends with a frequency of 1,400 Hz. A second string 40 cm long which has 5 nodes between its two ends (not counting the nodes at either end) is vibrating at a frequency of 1,440 Hz. What is the length of the first string which is vibrating with 7 antinodes? A third string which is 64 cm long is vibrating at a frequency of 900 Hz. How many nodes would this third string have between its two ends (not counting the nodes at either end)? (Assume that the three strings are similar in all respects except length.)
   Answer: The first string has 7 antinodes so it is vibrating at its 7th harmonic, and its fundamental frequency must be equal to 1,400 Hz divided by 7 i.e. 200 Hz. Since the second string is vibrating with 5 nodes it must have 6 antinodes and is vibrating at its 6th harmonic, so its fundamental frequency is equal to 1,440 Hz divided by 6 i.e. 240 Hz. The length of the first string is thus given by 40 cm times $\frac{240}{200}$ i.e. 48 cm. The third string is 64 cm long, so its fundamental frequency is equal to 240 Hz times $\frac{40}{64}$ i.e. 150 Hz. Since the third string is vibrating at a frequency of 900 Hz, it must be vibrating at its 6th harmonic as 900 Hz divided by 150 Hz is equal to 6. Therefore the third string must have 6 antinodes and 5 nodes between its two ends (not counting the nodes at either end).

2. A sailing boat is travelling in the same direction and speed as the water waves on the surface of a lake, and the length of the boat happens to be exactly equal to 6 complete wavelengths of the waves. Calculate the length of the boat if the waves are moving with a speed of 1.4 m/s with a frequency of 1 Hz. The speed of the waves then increases to 1.8 m/s and the frequency increases to 1.2 Hz. How many wavelengths would now equal the length of the boat?
Answer: The water waves have a wavelength equal to 1.4 m/s divided by 1 Hz i.e. 1.4 m, so the length of the boat is equal to 1.4 m times 6 i.e. 8.4 m. When the frequency of the waves increases to 1.2 Hz and the speed of the waves increases to 1.8 m/s, the wavelength of the waves is then equal to 1.8 m/s divided by 1.2 Hz i.e. 1.5 m. The number of wavelengths which would exactly equal the length of the boat is then given by 8.4 m divided by 1.5 m i.e. 5.6 wavelengths.

3. A string has a fundamental frequency of 126 Hz. Its frequency when vibrating with 8 antinodes between its two ends is the same as that of a closed pipe of length p cm. If the closed pipe is vibrating with 4 nodes between its two ends (not counting the node at one end), what is its fundamental frequency? When the closed pipe vibrates with 6 nodes between its two ends (not counting the node at one end), its frequency is the same as that of an open pipe vibrating with 6 antinodes between its two ends (not counting the antinodes at both ends). Calculate the length of

the open pipe.

Answer: Since the string is vibrating with 8 antinodes, it is at its 8th harmonic, so its frequency of vibration is equal to 126 Hz times 8 i.e. 1,008 Hz. The closed pipe has 4 nodes so it is at its 9th harmonic and its fundamental frequency is thus given by 1,008 Hz divided by 9 i.e. 112 Hz. When the closed pipe has 6 nodes, it will be at its 13th harmonic, so its frequency of vibration will be equal to 112 Hz times 13 i.e. 1,456 Hz. Since the open pipe has 6 antinodes it will have 7 nodes and will be at its 7th harmonic. Therefore its fundamental frequency is given by 1,456 Hz divided by 7 i.e. 208 Hz. An open pipe with the same length p cm as the closed pipe would have a fundamental frequency double that of the closed pipe i.e. 224 Hz, so the open pipe having a fundamental frequency of 208 Hz has a length equal to p cm times $\frac{224}{208}$ i.e. $\frac{14p}{13}$ cm.

4. A 42 cm long string vibrating with 3 nodes between its two ends (not counting the nodes at both ends) produces a musical note which combines with the note from a closed pipe. When the closed pipe which has a fundamental frequency of 66 Hz vibrates with 2 nodes between its two ends (not counting the node at one end), beats of 10 Hz are produced. When the string is shortened slightly, the beat frequency decreases (without passing through 0 Hz). Calculate the fundamental frequency of the string. The string is then shortened from 42 cm to 40 cm, and assuming that the beats are still produced by the

same harmonics of the string and the closed pipe as before, calculate the new beat frequency. If the closed pipe's length is decreased to 75% of its original length, what would the beat frequency then be, assuming that the string is still 40 cm long?

Answer: The closed pipe has 2 nodes so it is at its 5th harmonic, and its frequency is hence equal to 66 Hz times 5 i.e. 330 Hz. When the string is shortened slightly, its frequency will increase, so if the beat frequency decreases, the frequency of the string must have been lower than that of the closed pipe. Therefore when the beat frequency was 10 Hz, the frequency of the string was equal to 330 Hz minus 10 Hz i.e. 320 Hz. Since the string has 3 nodes and 4 antinodes, it is at its 4th harmonic and its fundamental frequency is thus equal to 320 Hz divided by 4 i.e. 80 Hz. Since the shortened string has a length of 40 cm, its fundamental frequency would be equal to 80 Hz times $\frac{42}{40}$ i.e. 84 Hz. The string's 4th harmonic would then be 84 Hz times 4 i.e 336 Hz so the beat frequency would become 336 Hz minus 330 Hz i.e. 6 Hz. When the length of the closed pipe is decreased to 75% of its original length, its fundamental frequency would change to 66 Hz times $\frac{1}{0.75}$ i.e. 88 Hz. Hence its 5th harmonic would be given by 88 Hz times 5 i.e. 440 Hz, and the beat frequency would change to 440 Hz minus 336 Hz i.e. 104 Hz.

5. The A string of a violin is being tuned by a violinist with the help of an electronic tuner which is producing a musical note with a frequency of 440 Hz. When

the note from the tuner combines with the note from the A string, beats of 4 Hz can be heard. When the violinist gradually loosens the A string of the violin, the beat frequency gradually decreases (without passing through 0 Hz) to 3 Hz. What was the frequency of the note produced by the violin's A string when the beat frequency was equal to 4 Hz? How can the violinist get the frequency of the A string to come as close as possible to 440 Hz? If the beat frequency had increased to 5 Hz instead on loosening the A string, determine the A string's frequency when the beat frequency was 4 Hz.

Answer: The beat frequency was 4 Hz, so the frequency of the A string's note was either 440 Hz minus 4 Hz i.e. 336 Hz, or 440 Hz plus 4 Hz i.e. 444 Hz. When the violin's A string was loosened, its frequency would have decreased, so if the beat frequency then decreased to 3 Hz, the frequency of the A string's note must have moved closer to 440 Hz. Therefore the A string's frequency must have been higher than 440 Hz when the beat frequency was 4 Hz, so the A string's frequency must have been equal to 444 Hz. To get the frequency of the A string to come even closer to 440 Hz, the violinist should continue to loosen the A string to make the beat frequency decrease further. When the beat frequency becomes zero Hz, the frequency of the A string will then be equal to 440 Hz. If the beat frequency had increased to 5 Hz on loosening the A string, the frequency of the violin's note would have been lower than 440 Hz, and therefore it must be 436 Hz when the beat frequency was 4 Hz.

6. A closed pipe labelled E is sliced into eight pieces of equal length, creating seven short open pipes labelled E1 to E7, and a short closed pipe labelled E8. Five of the short open pipes E1, E2, E3, E4 and E5 are joined up to make an open pipe labelled F, while the remaining short open pipes E6 and E7 are joined up with E8 to make a closed pipe labelled G. If the fundamental frequency of E was f Hz, calculate the fundamental frequencies of the pipes E8, F and G. Determine the ratio of the interval between the frequency of F when it vibrates with 6 nodes between its two ends and G when it vibrates with 5 nodes between its two ends (not counting the node at one end).
   Answer: The short closed pipe E8 is one-eighth the length of E so its fundamental frequency is equal to $8f$ Hz. The closed pipe G is three times the length of E8, so its fundamental frequency will be equal to $\frac{8f}{3}$ Hz, and when G has 5 nodes, it will be at its 11th harmonic, and its frequency will be given by $\frac{8f}{3}$ Hz times 11 i.e. $\frac{88f}{3}$ Hz. A closed pipe which has a length five times the length of E8 will have a fundamental frequency of $\frac{8f}{5}$ Hz, so F which is an open pipe of the same length will have a fundamental frequency double this i.e. $\frac{16f}{5}$ Hz. If F has 6 nodes it will be at its 6th harmonic and its frequency will then be given by $\frac{16f}{5}$ Hz times 6 i.e. $\frac{96f}{5}$ Hz. The ratio of the interval between $\frac{88f}{3}$ Hz and $\frac{96f}{5}$ Hz is given by $\frac{88f}{3}$

Hz divided by $\frac{96f}{5}$ Hz i.e. $\frac{55}{36}$.

Scientific Inquiry discussion points

The Equal-tempered scale is obtained by dividing an octave into twelve equal steps. This gives us a scale with twelve notes, which has become the basis of most of Western music, whether classical, popular, folk, rock or any other genre of music. The democratic equality of these twelve notes enables music to modulate into any of the twelve available keys with ease, using just twelve notes in one octave. This greatly simplifies the design of musical instruments and how they are played. But this is not a "perfect" system in mathematical terms, as the important interval of the fifth is not exactly 3/2 as in the Just and Pythagorean scales, but deviates by ever so slightly an amount which is not apparent to most listeners. Are there other examples in science and technology where imperfections are an important part of the basis of an actual working system?

While we may consider mathematical and physical perfection to be most desirable, in the real world most things and processes deviate from mathematical perfection. One crucial example is in the DNA of our genetic code. The reproduction of DNA as it replicates in the multiplication of living cells is not perfect, in that errors may occur in the replication due to natural events such as the alteration of the DNA code by natural radiation or cosmic rays. This may seem undesirable and it does lead to undesirable effects sometimes, but this same process makes evolution possible, as the errors in repli-

cation allow changes in the make-up of livings things, which may then make an organism less or more suited to the changing environment. Hence the progress brought about by evolution depends on these imperfections in replication.