

Answers to Tutorial No 3, Semester 1, 2026/27

1. A string is vibrating at a frequency of 1,600 Hz with 4 antinodes between its two ends. A second string 40 cm long which has 7 nodes between its two ends (not counting the nodes at either end) is vibrating at a frequency of 2,000 Hz. Calculate the length of the first string which is vibrating with 4 antinodes. A third string which is 50 cm long is vibrating at a frequency of 1,400 Hz. How many nodes would this third string have between its two ends (not counting the nodes at either end)? (Assume that the three strings are similar in all respects except length.)

Answer: Since the first string has 4 antinodes, it is vibrating at its 4th harmonic so its fundamental frequency is equal to 1,600 Hz divided by 4 i.e. 400 Hz. The second string is vibrating with 7 nodes so it must have 8 antinodes and is vibrating at its 8th harmonic. Its fundamental frequency is thus equal to 2,000 Hz divided by 8 i.e. 250 Hz. Therefore the length of the first string is given by 40 cm times $\frac{250}{400}$ i.e. 25 cm. The third string is 50 cm long, so its fundamental frequency is equal to 250 Hz times $\frac{40}{50}$ i.e. 200 Hz. The third string is vibrating at a frequency of 1,400 Hz, so it must be vibrating at its 7th harmonic as 1,400 Hz divided by 200 Hz is equal to 7. Therefore the third string must have 7 antinodes and 6 nodes between its two ends (not counting the nodes at either end).

2. A yacht is travelling in the same direction and speed as the water waves on the surface of the sea. The length of the yacht is exactly equal to 8 complete wavelengths of the waves. What is the length of the yacht if the waves are moving with a speed of 1.8 m/s with a frequency of 1.2 Hz? If the speed of the waves then decreases to 1.2 m/s and the frequency decreases to 1.0 Hz, how many wavelengths would then equal the length of the yacht?

Answer: Since the sea waves have a wavelength equal to 1.8 m/s divided by 1.2 Hz i.e. 1.5 m, the length of the yacht is equal to 1.5 m times 8 i.e. 12 m. When the frequency of the waves decreases to 1.0 Hz and the speed of the waves increases to 1.2 m/s, the wavelength of the waves is then equal to 1.2 m/s divided by 1.0 Hz i.e. 1.2 m. Therefore the number of wavelengths which would exactly equal the length of the yacht is given by 12 m divided by 1.2 m i.e. 10 wavelengths.

3. A string is vibrating with 6 antinodes between its two ends and has a fundamental frequency of 140 Hz. Its frequency of vibration is the same as that of a closed pipe of length k cm which is vibrating with 3 nodes between its two ends (not counting the node at one end). Calculate the fundamental frequency of the closed pipe. When the closed pipe vibrates with 4 nodes between its two ends (not counting the node at one end), its frequency is the same as that of an open pipe vibrating with 5 antinodes between its two ends (not counting the antinodes at both ends).

What is the length of the open pipe?

Answer: The string is vibrating with 6 antinodes so it is at its 6th harmonic and its frequency of vibration is equal to 140 Hz times 6 i.e. 840 Hz. Since the closed pipe has 3 nodes, it is at its 7th harmonic and its fundamental frequency is equal to 840 Hz divided by 7 i.e. 120 Hz. When the closed pipe has 4 nodes, it will be at its 9th harmonic and its frequency of vibration will be given by 120 Hz times 9 i.e. 1,080 Hz. The open pipe has 5 antinodes so it will have 6 nodes and will be at its 6th harmonic. Therefore its fundamental frequency is given by 1,080 Hz divided by 6 i.e. 180 Hz. Since an open pipe with the same length k cm as the closed pipe would have a fundamental frequency double that of the closed pipe i.e. 240 Hz, the open pipe which has a fundamental frequency of 180 Hz would have a length given by k cm times $\frac{240}{180}$ i.e. $\frac{4k}{3}$ cm.

4. A string which is vibrating with 4 nodes between its two ends (not counting the nodes at both ends) and is 54.5 cm long produces a musical note which combines with a note from a closed pipe. The closed pipe which has a fundamental frequency of 48 Hz vibrates with 5 nodes between its two ends (not counting the node at one end), producing beats of 11 Hz with the vibrating string. When the string is slightly shortened, the beat frequency increases (without passing through 0 Hz). What is the fundamental frequency of the string? If the string is shortened from 54.5 cm to 53.9 cm, and assuming that the beats are still

produced by the same harmonics of the string and the closed pipe as before, calculate the new beat frequency. If the closed pipe's length is increased to 120% of its original length, calculate the beat frequency assuming that the string is still 53.9 cm long.

Answer: Since the closed pipe has 5 nodes, it is at its 11th harmonic, and its frequency is given by 48 Hz times 11 i.e. 528 Hz. When the string is shortened slightly, its frequency will increase. Hence if the beat frequency increases, the frequency of the string must have been higher than that of the closed pipe. So when the beat frequency was 11 Hz, the frequency of the string was equal to 528 Hz plus 11 Hz i.e. 539 Hz. Since the string has 4 nodes and 5 antinodes, it is at its 5th harmonic and its fundamental frequency is thus equal to 539 Hz divided by 5 i.e. 107.8 Hz. Since the shortened string has a length of 53.9 cm, its fundamental frequency would be equal to 107.8 Hz times $\frac{54.5}{53.9}$ i.e. 109 Hz. The string's 5th harmonic would then be 109 Hz times 5 i.e. 545 Hz so the beat frequency would become 545 Hz minus 528 Hz i.e. 17 Hz. When the length of the closed pipe is increased to 120% of its original length, its fundamental frequency would change to 48 Hz times $\frac{100}{120}$ i.e. 40 Hz. Hence its 11th harmonic would be given by 40 Hz times 11 i.e. 440 Hz, and the beat frequency would change to 545 Hz minus 440 Hz i.e. 105 Hz.

5. A 'cellist tunes the A string of a 'cello with the help of an electronic tuner which is producing a musical note with a frequency of 220 Hz. When the note

from the A string combines with the note from the tuner, beats of 3 Hz can be heard. When the 'cellist gradually tightens the A string of the 'cello, the beat frequency gradually decreases (without passing through 0 Hz) to 2 Hz. Calculate the frequency of the note produced by the 'cello's A string when the beat frequency was equal to 3 Hz. Explain how the 'cellist could get the frequency of the A string to come as close as possible to 220 Hz. If the beat frequency had increased to 4 Hz instead on tightening the A string, what would have been the A string's frequency when the beat frequency was 3 Hz?

Answer: Since the beat frequency was 3 Hz, the frequency of the A string's note was either 220 Hz minus 3 Hz i.e. 217 Hz, or 220 Hz plus 3 Hz i.e. 223 Hz. When the 'cello's A string was tightened, its frequency would have increased. Hence if the beat frequency then decreased to 2 Hz, the frequency of the A string's note must have moved closer to 220 Hz. Therefore the A string's frequency must have been lower than 220 Hz when the beat frequency was 3 Hz, which means that the A string's frequency must have been equal to 217 Hz. In order to get the frequency of the A string to come even closer to 220 Hz, the 'cellist should continue to tighten the A string to make the beat frequency decrease further, and when the beat frequency becomes zero Hz, the frequency of the A string will then be exactly equal to 220 Hz. If the beat frequency had increased to 4 Hz on tightening the A string, the frequency of the 'cello's note must have been higher than 220 Hz. Its

frequency must therefore be 223 Hz when the beat frequency was 3 Hz.

6. A closed pipe labelled A is sliced into seven pieces of equal length. This creates six short open pipes labelled A1 to A6, and a short closed pipe labelled A7. Four of the short open pipes A1, A2, A3 and A4 are joined up to make an open pipe labelled B. The remaining short open pipes A5 and A6 are joined up with A7 to make a closed pipe labelled C. If the fundamental frequency of A was f Hz, what would be the fundamental frequencies of the pipes A1, B and C? Calculate the ratio of the interval between the frequency of B when it vibrates with 7 nodes between its two ends and C when it vibrates with 6 nodes between its two ends (not counting the node at one end).

Answer: The short open pipe A1 is one-seventh the length of A. A closed pipe of this length would have a fundamental frequency equal to $7f$ Hz, so A1 being an open pipe has a fundamental frequency double this i.e. $14f$ Hz. The open pipe B is four times the length of A1, so its fundamental frequency will be equal to one-quarter that of A1 i.e. $\frac{7f}{2}$ Hz. When B has 7 nodes, it will be at its 7th harmonic, and its frequency will be given by $\frac{7f}{2}$ Hz times 7 i.e. $\frac{49f}{2}$ Hz. The closed pipe C has a length three-sevenths the length of A, so it will have a fundamental frequency of $\frac{7f}{3}$ Hz. If C has 6 nodes it will be at its 13th harmonic and its frequency will then be given by $\frac{7f}{3}$ Hz

times 13 i.e. $\frac{91f}{3}$ Hz. The ratio of the interval between $\frac{91f}{3}$ Hz and $\frac{49f}{2}$ Hz is equal to $\frac{91f}{3}$ Hz divided by $\frac{49f}{2}$ Hz i.e. $\frac{182}{147}$.

Scientific Inquiry discussion points

The Equal-tempered scale is obtained by dividing an octave into twelve equal steps. This gives us a scale with twelve notes, which has become the basis of most of Western music, whether classical, popular, folk, rock or any other genre of music. The democratic equality of these twelve notes enables music to modulate into any of the twelve available keys with ease, using just twelve notes in one octave. This greatly simplifies the design of musical instruments and how they are played. But this is not a “perfect” system in mathematical terms, as the important interval of the fifth is not exactly $3/2$ as in the Just and Pythagorean scales, but deviates by ever so slightly an amount which is not apparent to most listeners. Are there other examples in science and technology where imperfections are an important part of the basis of an actual working system?

While we may consider mathematical and physical perfection to be most desirable, in the real world most things and processes deviate from mathematical perfection. One crucial example is in the DNA of our genetic code. The reproduction of DNA as it replicates in the multiplication of living cells is not perfect, in that errors may occur in the replication due to natural events such as the alteration of the DNA code by natural radiation or cosmic rays. This may seem undesirable and it does

lead to undesirable effects sometimes, but this same process makes evolution possible, as the errors in replication allow changes in the make-up of living things, which may then make an organism less or more suited to the changing environment. Hence the progress brought about by evolution depends on these imperfections in replication.